# Linear Classifiers

Jaime Arguello
INLS 613: Text Data Mining
jarguell@email.unc.edu

# Overview

- Philosophical questions
- Derivatives: What are they good for?
- Linear regression
- Multiple linear regression
- Logistic regression

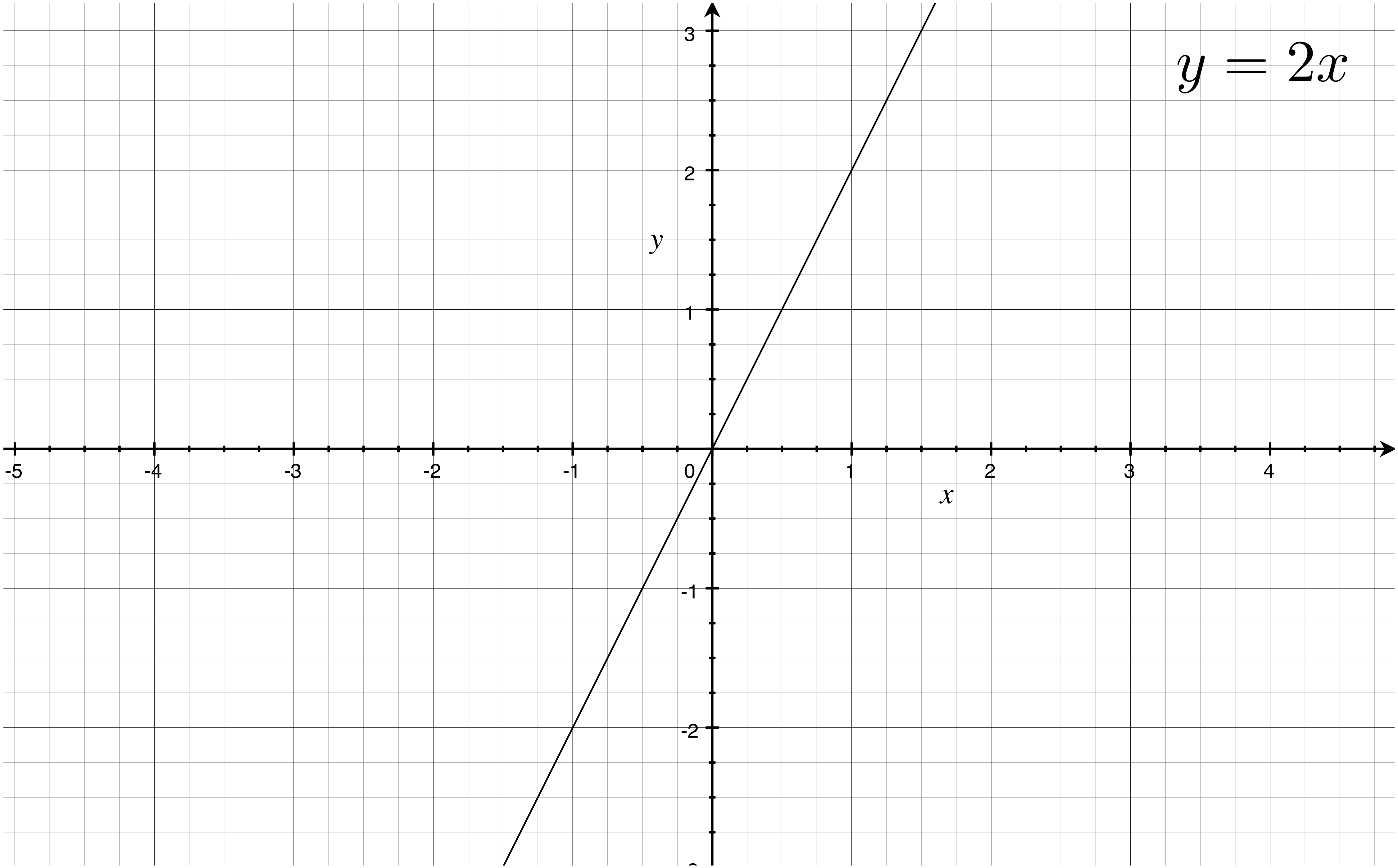
# Philosophical Questions

- What would you do if …
- What does this have to do with linear classifiers?

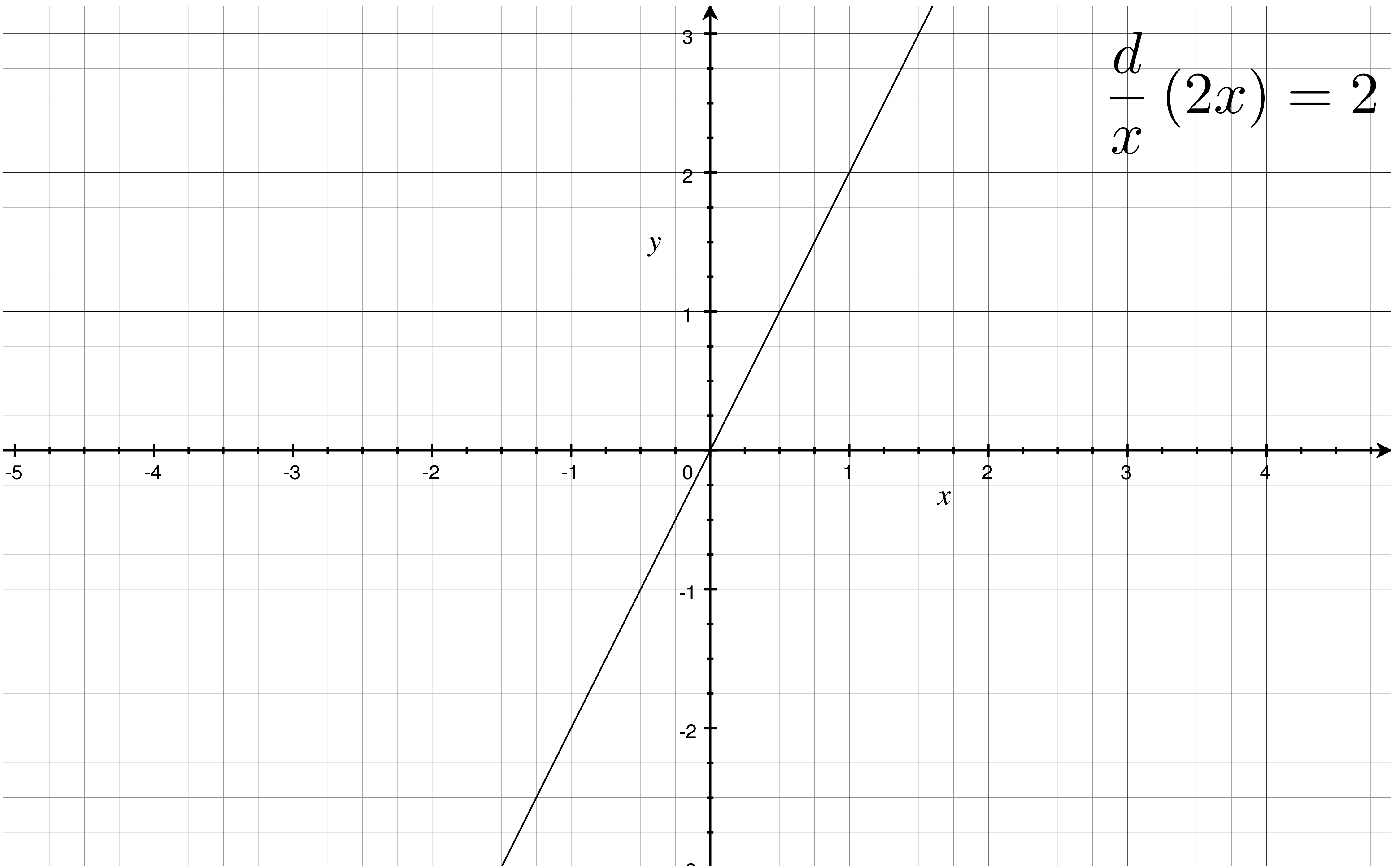
# Functions

$$x \longrightarrow \boxed{f(x)} \longrightarrow y$$

# Derivatives

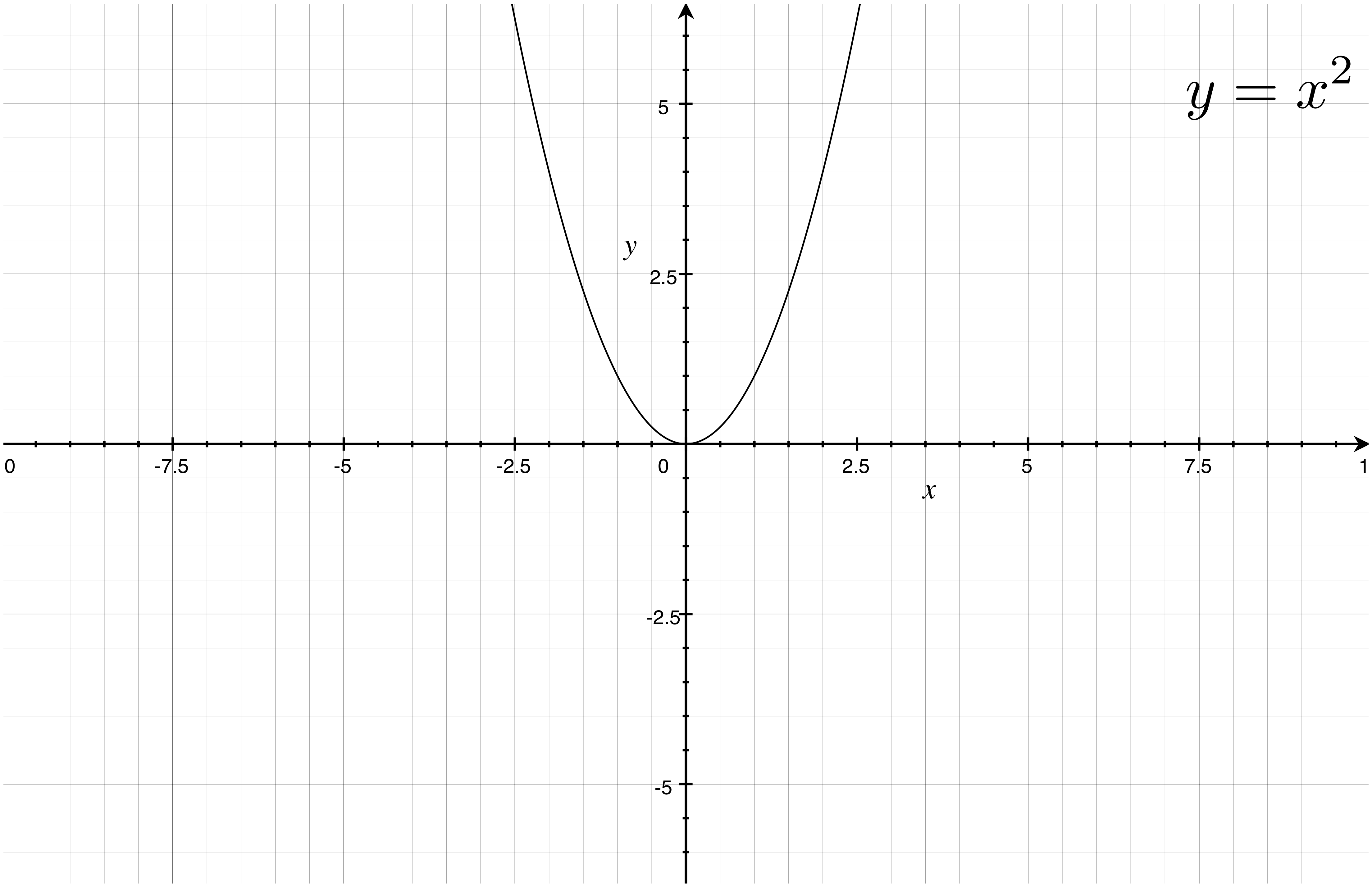


# Derivatives

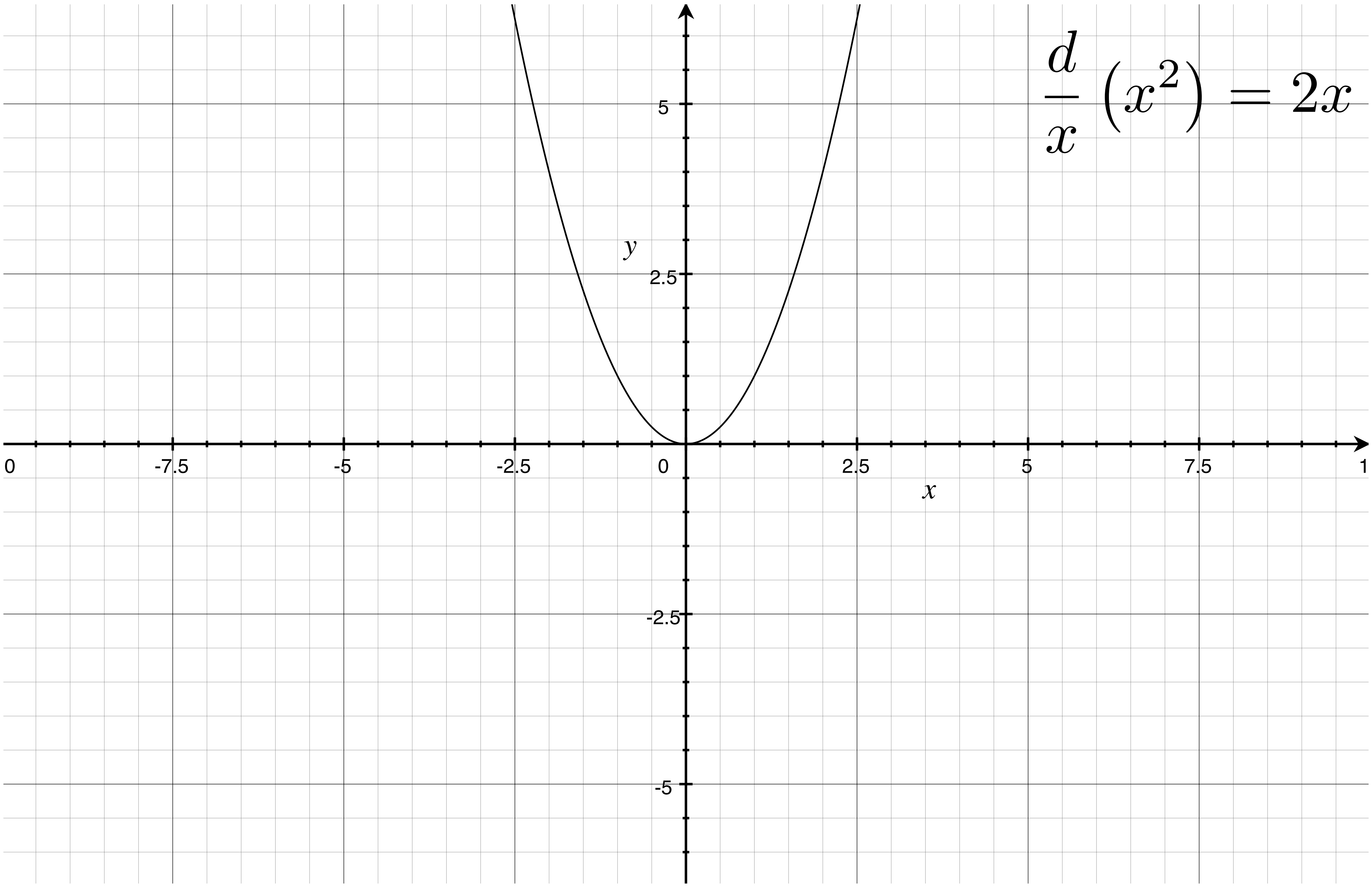



$$\frac{d}{x}(2x) = 2$$

# Derivatives



# Derivatives

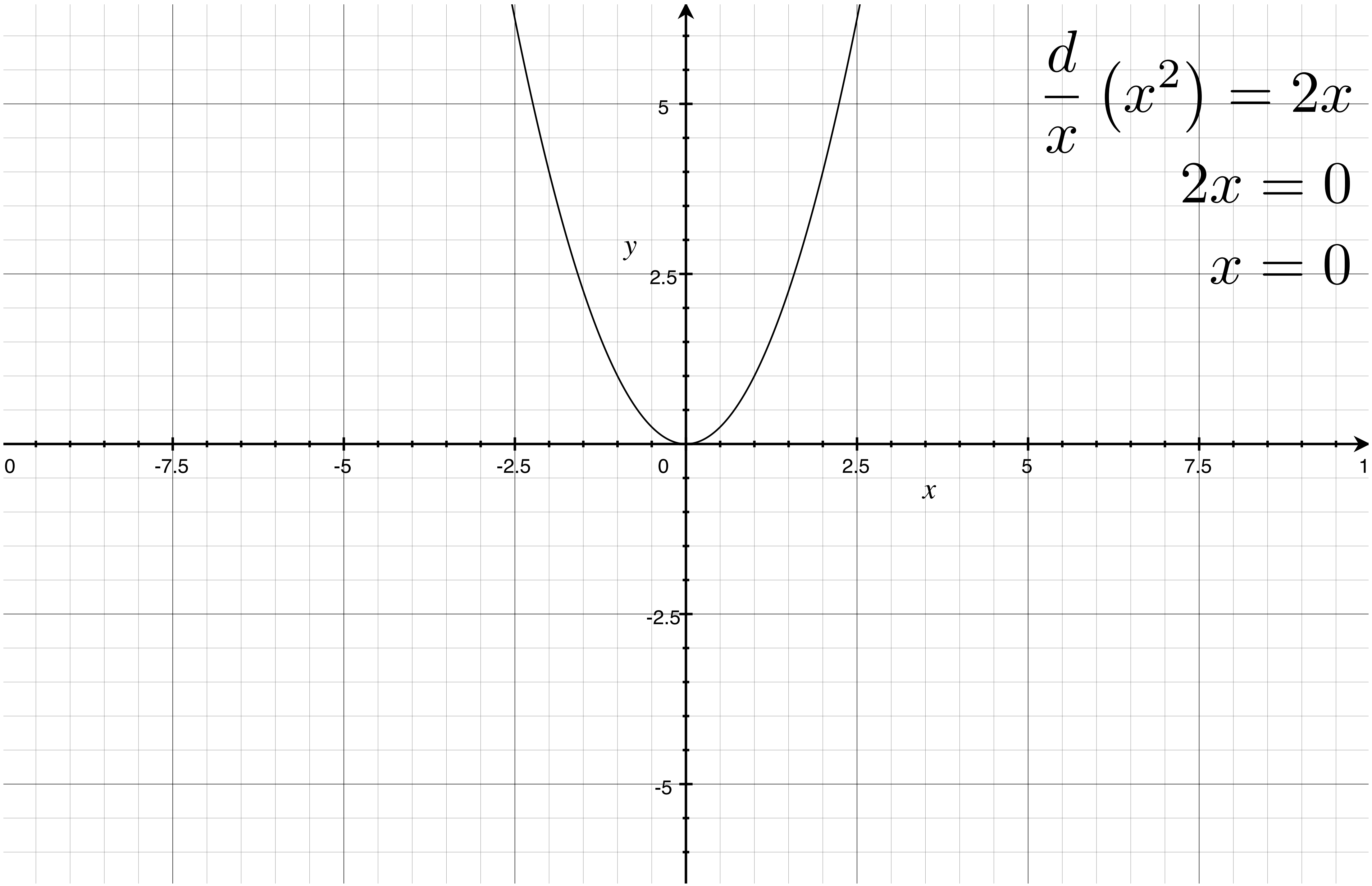



$$\frac{d}{x}\left(x^2\right) = 2x$$

# Derivatives: What are they good for?

- The derivative of f(x) outputs the slope of f(x) for a particular value of x
- A point of which the slope is zero is a point at which f(x) is at its highest or lowest value.
- What does this have to do with machine learning?

# Derivatives



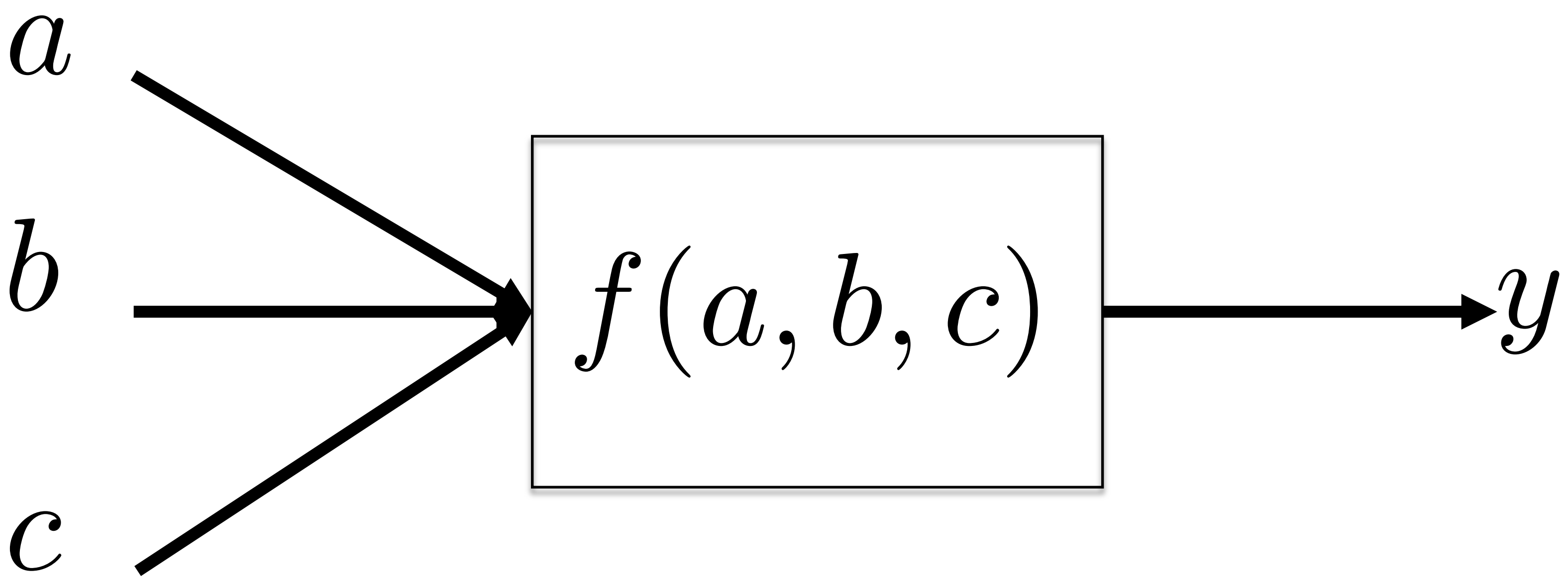

$$\frac{d}{x}\left(x^2\right) = 2x$$

$$2x = 0$$

$$x = 0$$

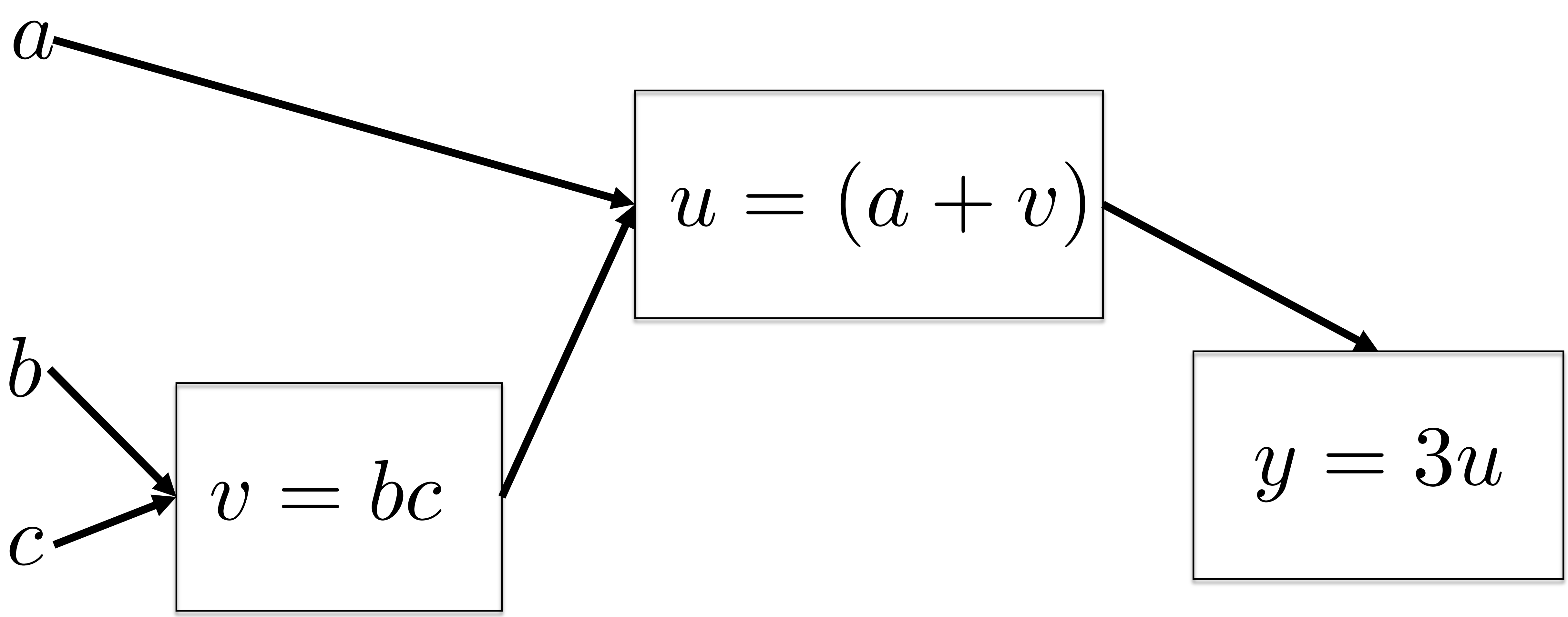
# Computation Graphs

$$y = 3(a + bc)$$



# Computation Graphs

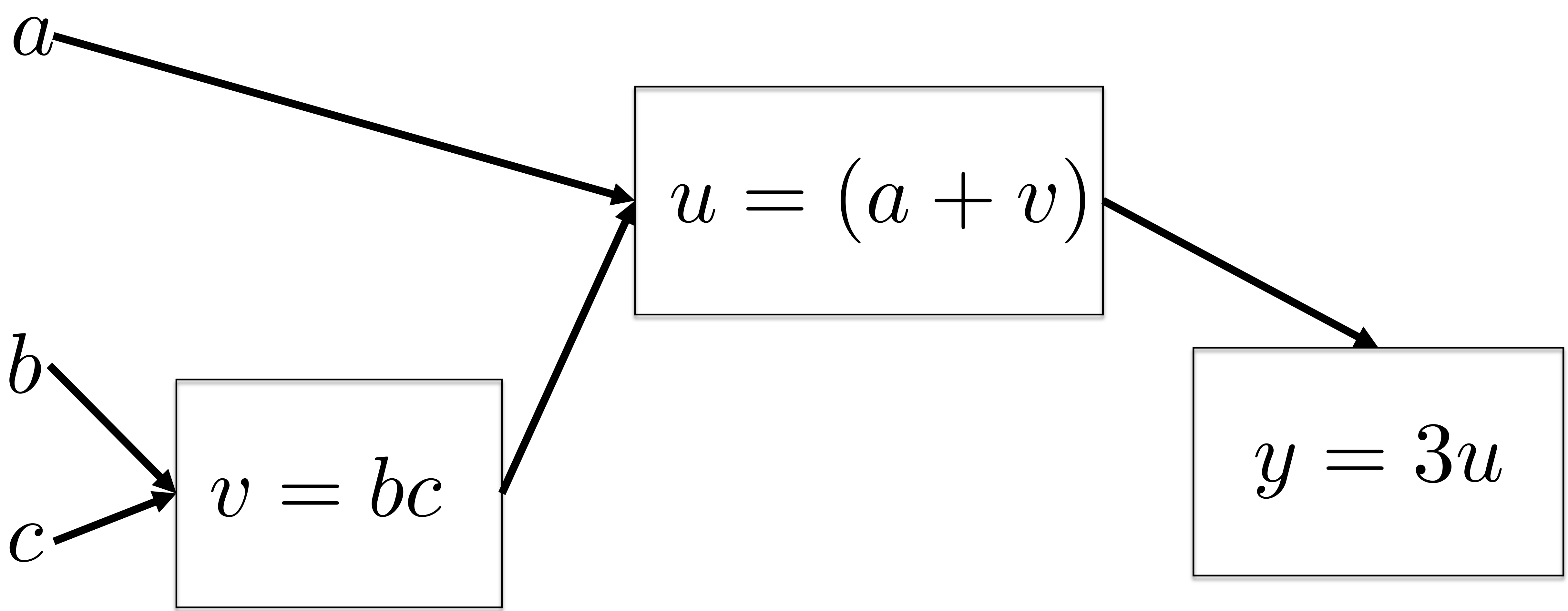
$$y = 3(a + bc)$$



# Derivatives: Chain Rule

$$y = 3(a + bc)$$

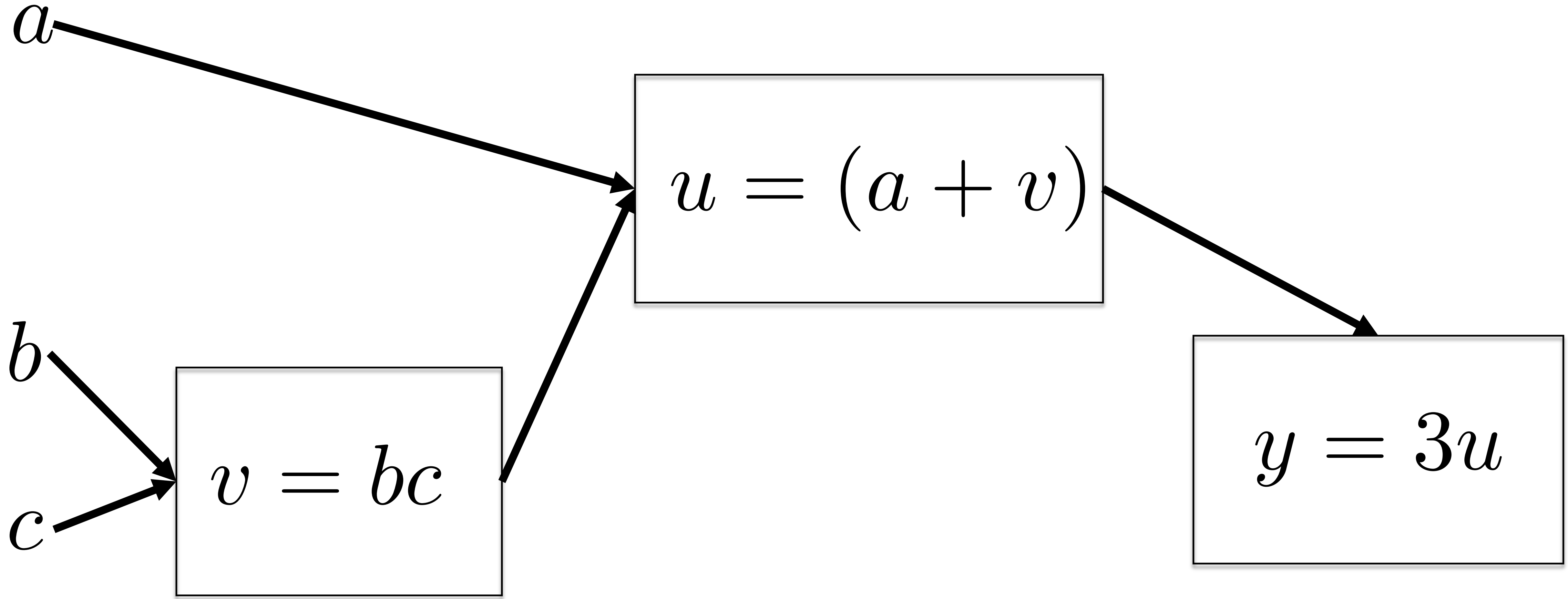
$$\frac{dy}{dc} = \frac{dv}{dc} \times \frac{du}{dv} \times \frac{dy}{du}$$



# Derivatives: Chain Rule

$$y = 3(a + bc)$$

$$\frac{dy}{dc} = b \times 1 \times 3 = 3b$$



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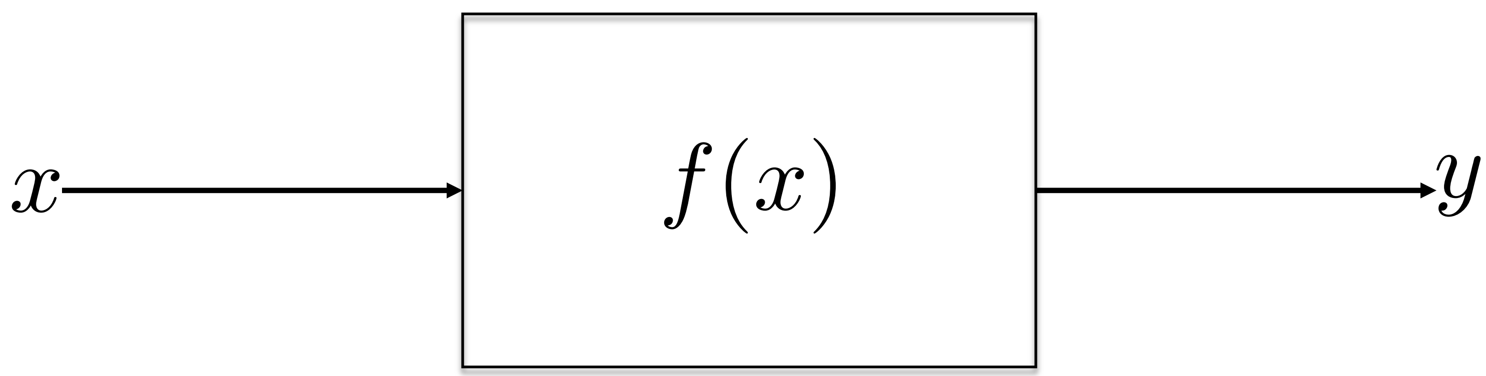
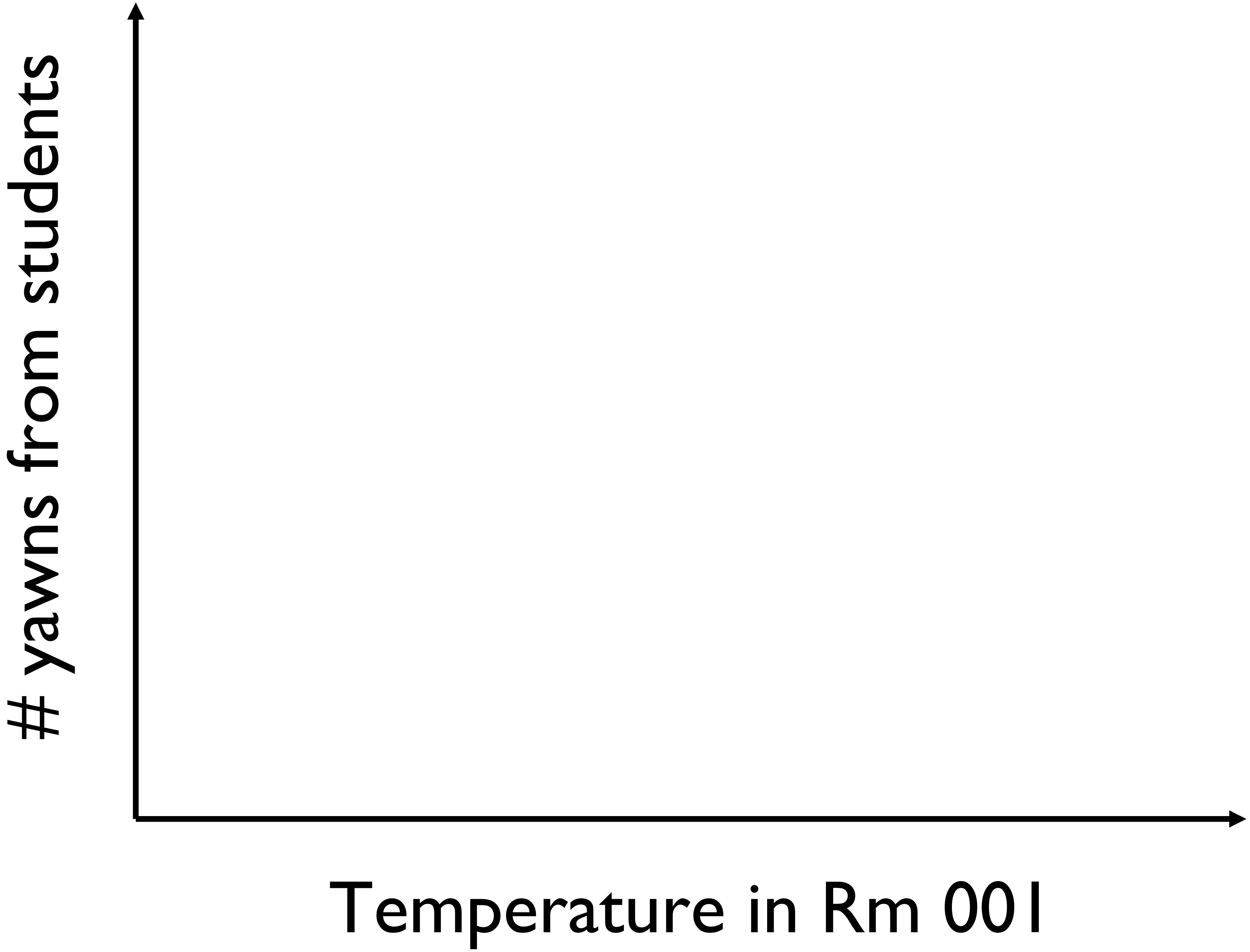
- Derivatives: What are they good for?
- Linear regression
- Multiple linear regression
- Logistic regression

# Linear Regression

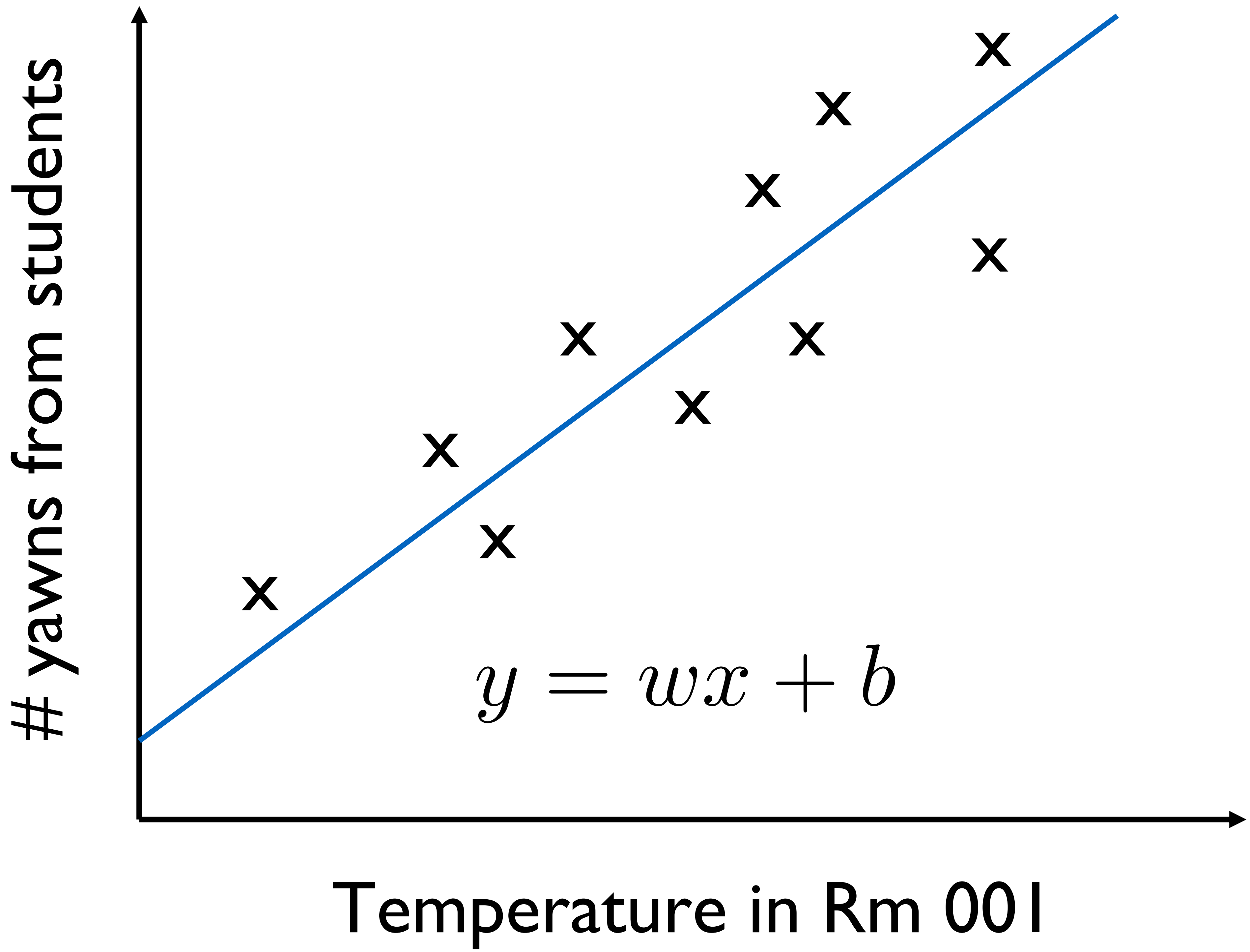



$$y = wx + b$$

# Linear Regression



# Linear Regression



# Linear Regression: Training

$$y = wx + b$$

- **Input:** set of *m* training examples (x,y)
- Find the value of *w* and *b* that minimize the error:

$$\sum_{i=1}^{m} \left( y^{(i)} - \hat{y}^{(i)} \right)^2$$

# Linear Regression: Training

$$y = wx + b$$

- Find the value of *w* and *b* that minimize the error:

$$\sum_{i=1}^{m} \left( y^{(i)} - \hat{y}^{(i)} \right)^2$$

$$\sum_{i=1}^{m} \left( y^{(i)} - wx^{(i)} - b \right)^2$$

# Linear Regression: Training

$$y = wx + b$$

- Find the value of *w* and *b* that minimize the error:

$$\sum_{i=1}^{m} \left( y^{(i)} - wx^{(i)} - b \right)^2$$

- Take the derivative with respect to *w*, set it equal to 0, and solve for *w*.
- Take the derivative with respect to *b*, set it equal to 0, and solve for *b*.

# Linear Regression: Training

- Find the value of $w$ and $b$ that minimize the error:

$$w = \frac{\frac{1}{m}\sum_{i=1}^{m}\left(x^{(i)} - \bar{x}\right)\left(y^{(i)} - \bar{y}\right)}{\sum_{i=1}^{m}\left(x^{(i)} - \bar{x}\right)^2}$$

$$b = \bar{y} - w\bar{x}$$

http://seismo.berkeley.edu/~kirchner/eps_120/Toolkits/Toolkit_10.pdf

# Linear Regression: Training

- Find the value of *w* and *b* that minimize the error:

$$w = \frac{\boxed{\frac{1}{m}\sum_{i=1}^{m}\left(x^{(i)} - \bar{x}\right)\left(y^{(i)} - \bar{y}\right)}}{\boxed{\sum_{i=1}^{m}\left(x^{(i)} - \bar{x}\right)^2}}$$

Always positive!

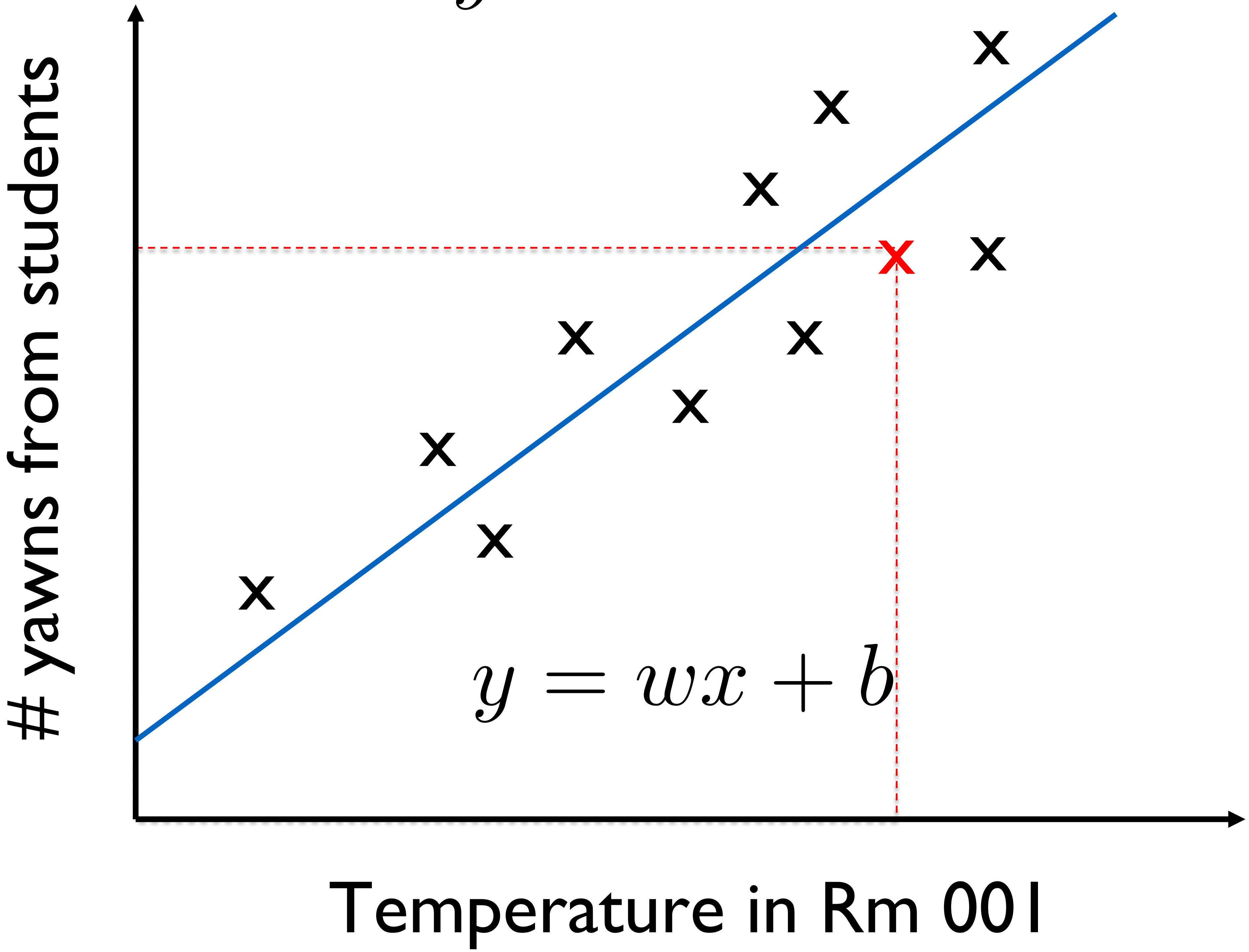
It depends!

$$b = \bar{y} - w\bar{x}$$

http://seismo.berkeley.edu/~kirchner/eps_120/Toolkits/Toolkit_10.pdf

# Linear Regression: Prediction

$$y = wx + b$$



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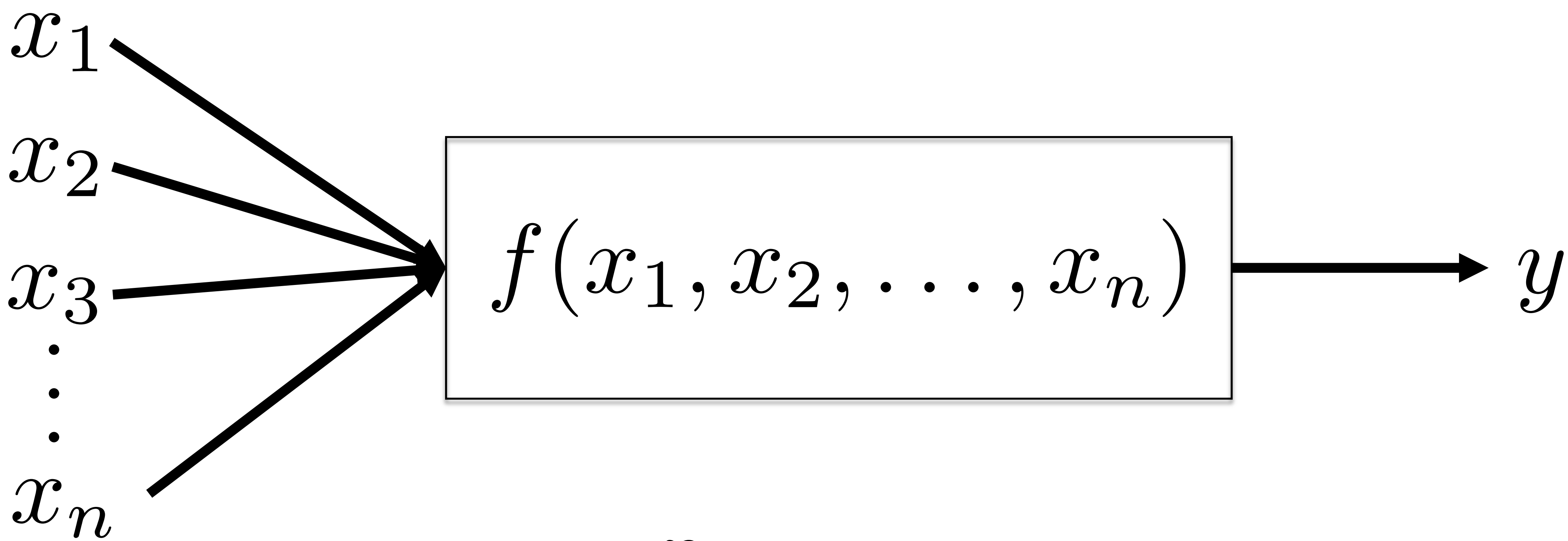
# Multiple Linear Regression




$$y = \sum_{j=1}^{n} (w_j x_j) + b$$

# Multiple Linear Regression

| Size (feet) | No. of bedrooms | No. of floors | Age (years) | Price (x$1000) |
| --- | --- | --- | --- | --- |
| 2,350 | 5 | 2 | 45 | 500 |
| 1,600 | 3 | 2 | 20 | 450 |
| 2,000 | 3 | 2 | 30 | 250 |
| 854 | 2 | 1 | 10 | 200 |
| 560 | 1 | 1 | 30 | 180 |

# Multiple Linear Regression: Training

- Given:

$$\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$$

- We want:

$$\hat{y}^{(i)} \approx y^{(i)}$$

# Multiple Linear Regression: Training

- Loss Function: the discrepancy between the predicted and actual output values for a single training instance

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2$$

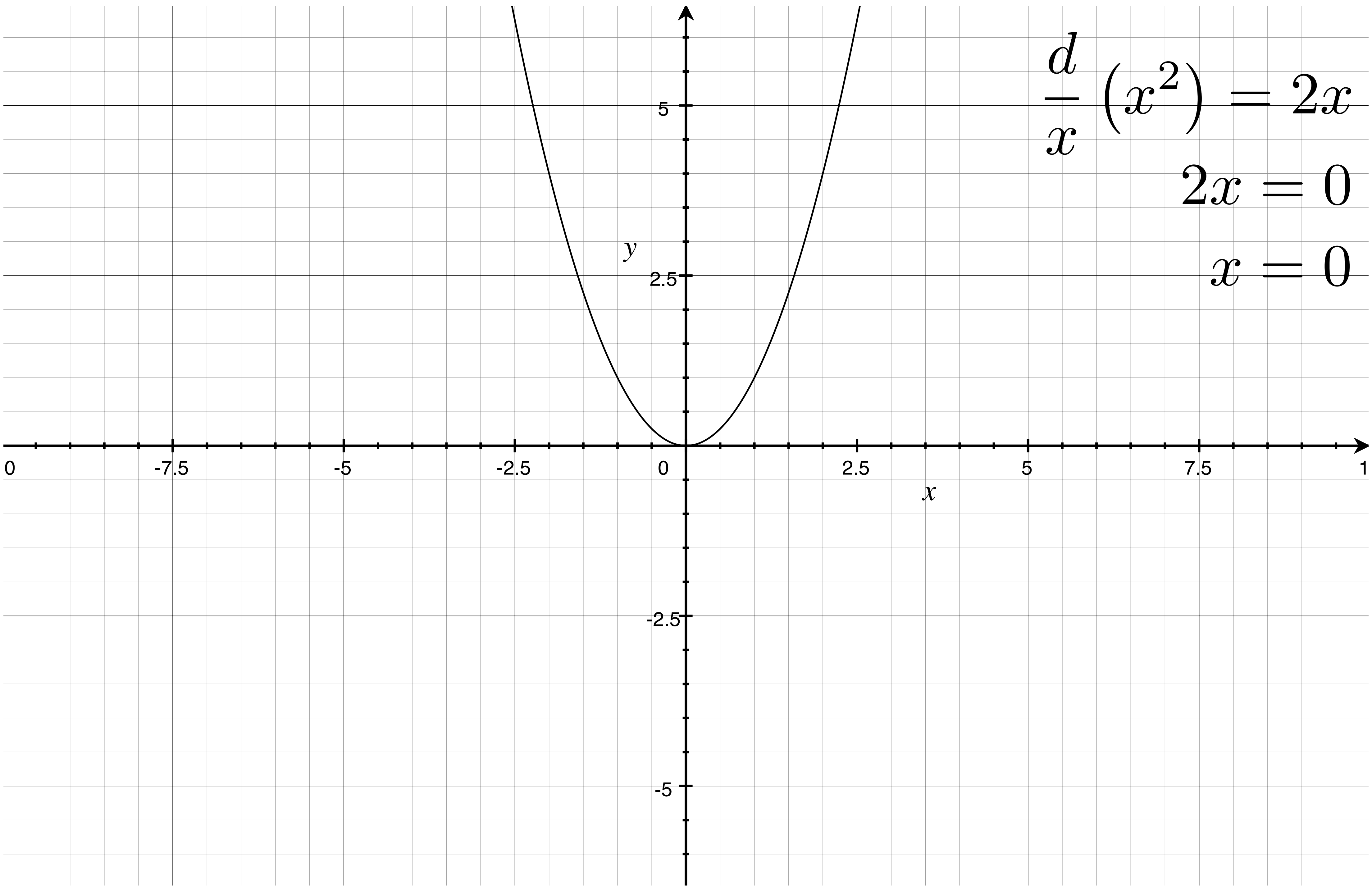
# Multiple Linear Regression: Training

- Cost Function: the discrepancy between the predicted and actual output values for all training instances

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2$$

$$\mathcal{J}(w, b) = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2\right)$$

# Derivatives

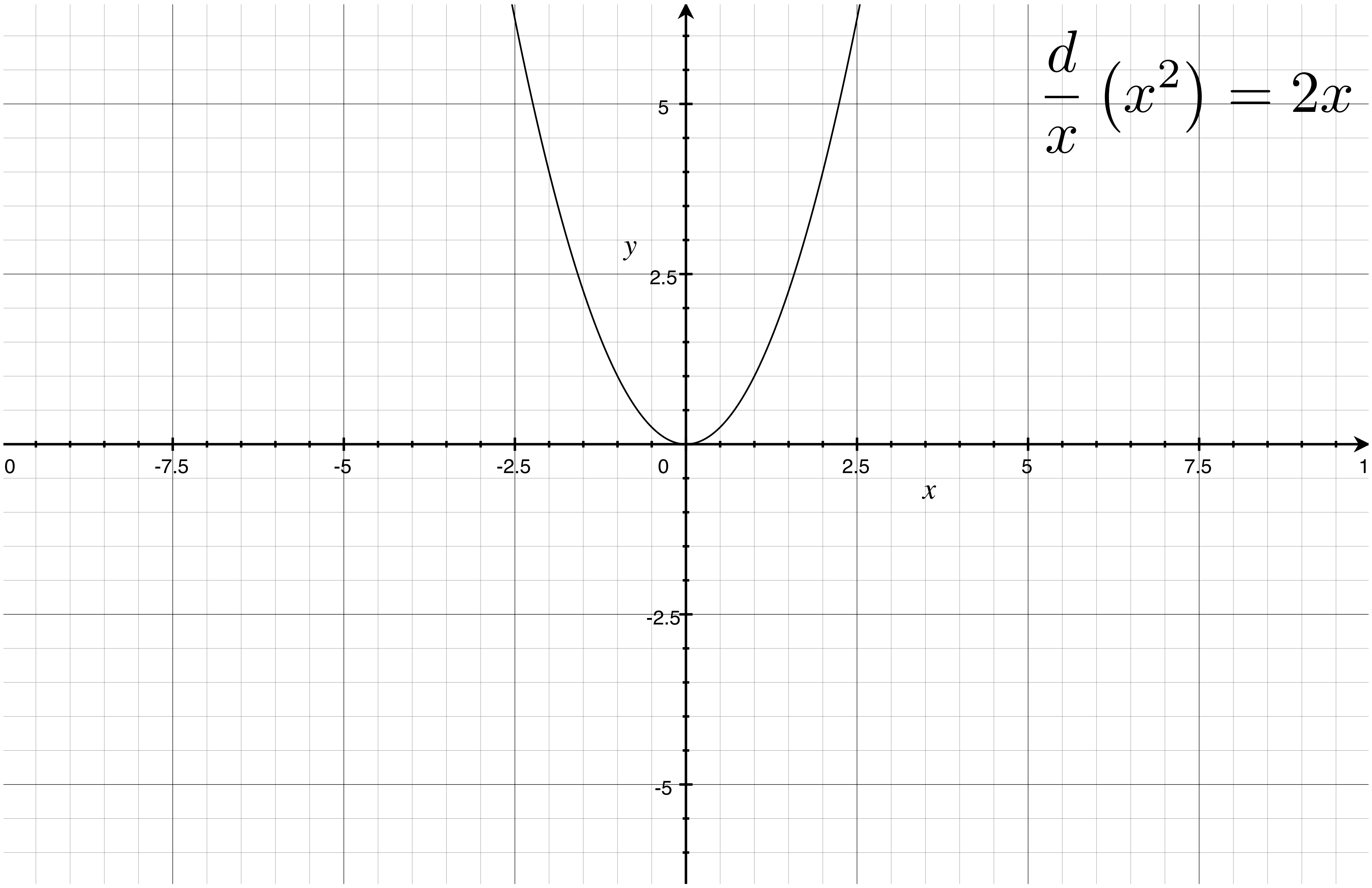



$$\frac{d}{x}\left(x^2\right) = 2x$$

$$2x = 0$$

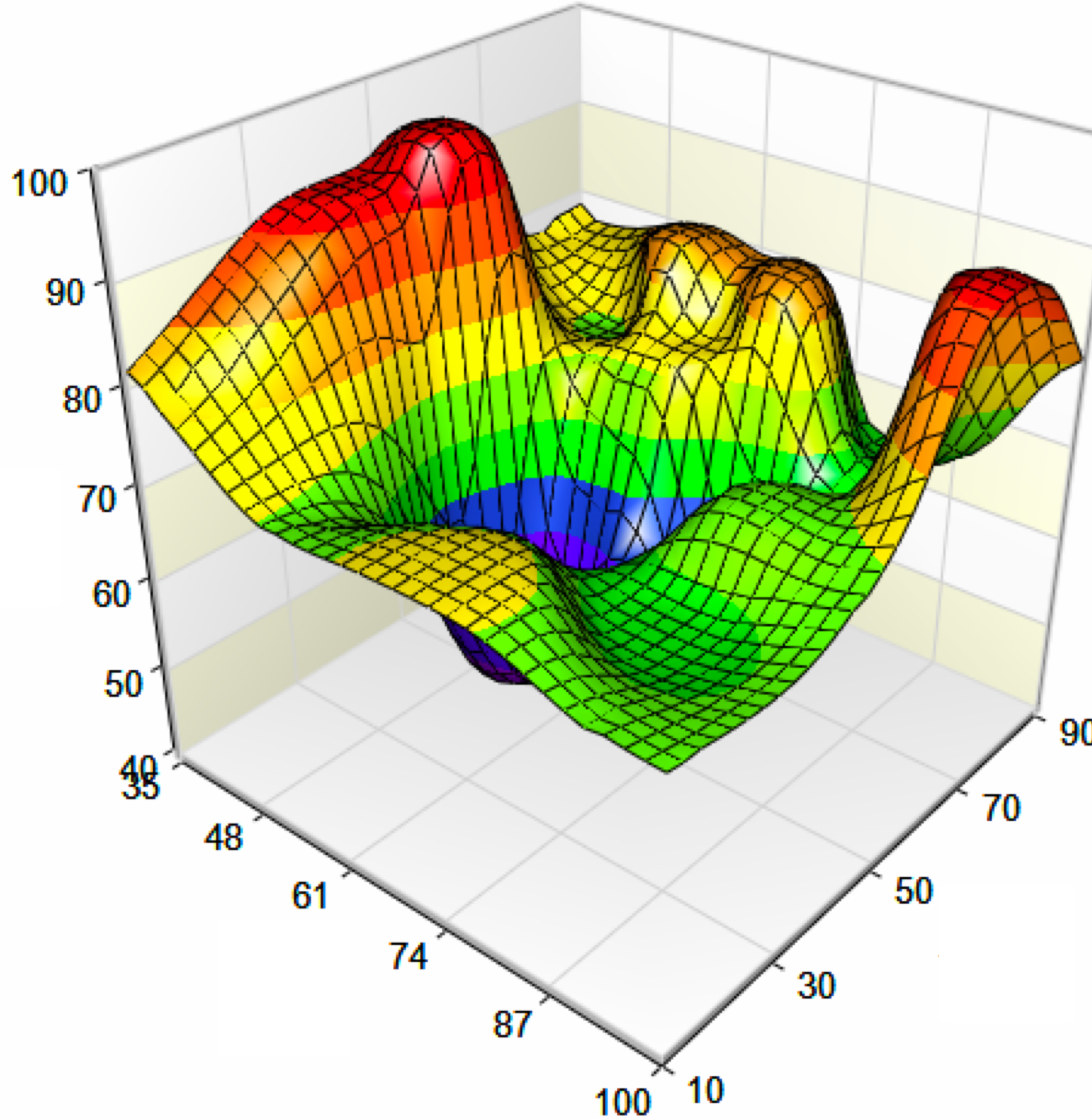
$$x = 0$$

# Gradient Descent: Intuition




$$\frac{d}{x}\left(x^2\right) = 2x$$

# Gradient Descent: Intuition



# Multiple Linear Regression

| Size (feet) | No. of bedrooms | No. of floors | Age (years) | Price (x$1000) |
|---|---|---|---|---|
| 2,350 | 5 | 2 | 45 | 500 |
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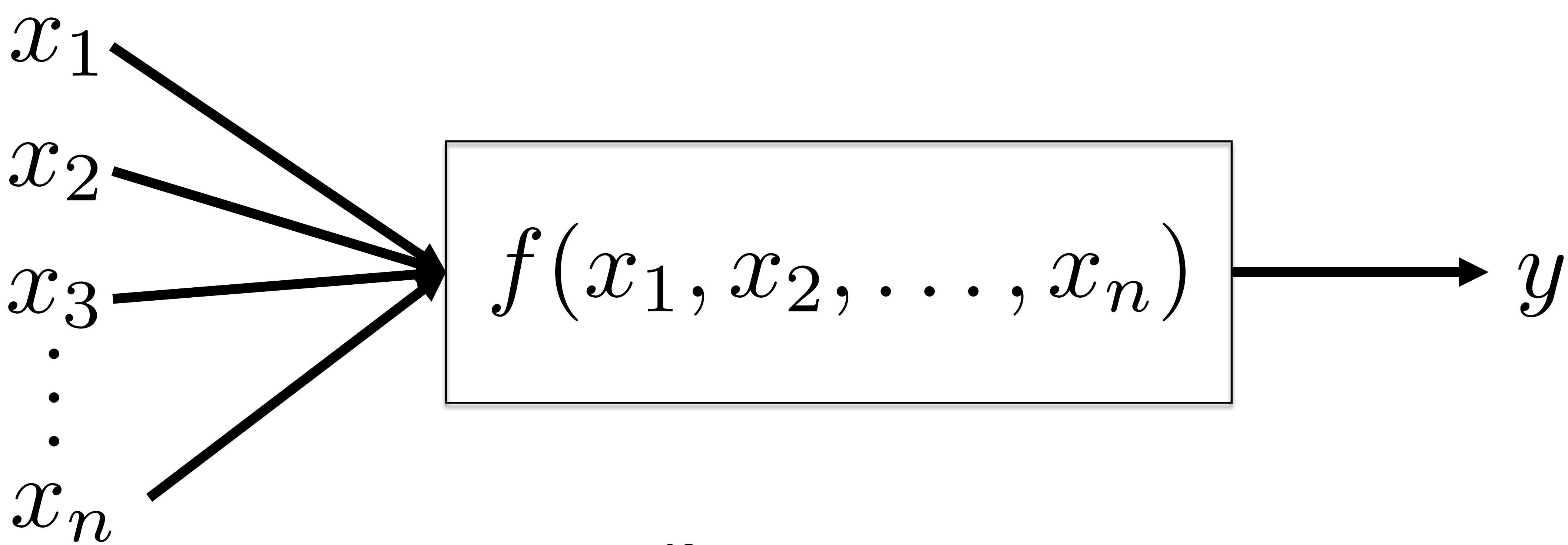
# Multiple Linear Regression




$$y = \sum_{j=1}^{n} (w_j x_j) + b$$

# Gradient Descent

- Loss Function: the discrepancy between the predicted and actual output values for a single training instance

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2$$

- Let's see what the slope of the loss function is with respect to parameter $b$!
- **Note:** this will only consider one training example!

# Gradient Descent

- Derivative of the loss function with respect to $b$

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2$$

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}\left(y^{(i)} - \sum_{j=1}^{n}(w_j x_j^{(i)}) - b\right)^2$$

$$\frac{d}{db}\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = -\left(y^{(i)} - \hat{y}^{(i)}\right)$$

$$\frac{d}{db}\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \hat{y}^{(i)} - y^{(i)}$$

# Gradient Descent

$$\frac{d}{db}\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \hat{y}^{(i)} - y^{(i)}$$

| Scenario | $\hat{y}^{(i)} - y^{(i)}$ | Action! |
|---|---|---|
| $\hat{y}^{(i)} > y^{(i)}$ | + | Decrease (nudge left) |
| $\hat{y}^{(i)} < y^{(i)}$ | -- | Increase (nudge right) |
| $\hat{y}^{(i)} \approx y^{(i)}$ | 0 | Do nothing! |

$$y = \sum_{j=1}^{n}(w_j x_j) + b$$

# Gradient Descent

- Loss Function: the discrepancy between the predicted and actual output values for a single training instance

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2$$

- Let's see what the slope of the loss function is with respect to parameter $w_j$!
- **Note:** this will only consider one training example!

# Gradient Descent

- Derivative of the loss function with respect to $w_j$

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2$$

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}\left(y^{(i)} - \sum_{j=1}^{n}(w_j x_j^{(i)}) - b\right)^2$$

$$\frac{d}{dw_j}\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = -x_j^{(i)}\left(y^{(i)} - \hat{y}^{(i)}\right)$$

$$\frac{d}{dw_j}\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = (\hat{y}^{(i)} - y^{(i)})x_j$$

# Gradient Descent

$$\frac{d}{dw_j}\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = (\hat{y}^{(i)} - y^{(i)})x_j$$

| Scenario | $\hat{y}^{(i)} - y^{(i)}$ | Action! |
|---|---|---|
| $\hat{y}^{(i)} > y^{(i)}$ | + | Go in the opposite direction as $x_j^{(i)}$ |
| $\hat{y}^{(i)} < y^{(i)}$ | -- | Go in the same direction as $x_j^{(i)}$ |
| $\hat{y}^{(i)} \approx y^{(i)}$ | 0 | Do nothing! |

$$y = \sum_{j=1}^{n}(w_j x_j) + b$$

# Gradient Descent

- Loss Function: the discrepancy between the predicted and actual output values for a single training instance

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2$$

- Given one training example, we can take derivatives with respect to each parameter to see what direction we should be going to minimize the loss function.

# Gradient Descent

- Repeat many times (or until convergence):

$$b \leftarrow b - \alpha \frac{1}{m} \sum_{i=1}^{m} \left( \hat{y}^{(i)} - y^{(i)} \right)$$

$$w_j \leftarrow w_j - \alpha \frac{1}{m} \sum_{i=1}^{m} \left( (\hat{y}^{(i)} - y^{(i)}) x_j^{(i)} \right)$$

# Gradient Descent

- Repeat many times (or until convergence):

$$b \leftarrow b - \alpha \frac{1}{m} \sum_{i=1}^{m} \left( \hat{y}^{(i)} - y^{(i)} \right)$$

- If we are overshooting the target, reduce $b$
- If we are undershooting the target, increase $b$
- Otherwise, do nothing

# Gradient Descent

- Repeat many times (or until convergence):

$$w_j \leftarrow w_j - \alpha \boxed{\frac{1}{m} \sum_{i=1}^{m} \left( (\hat{y}^{(i)} - y^{(i)}) x_j^{(i)} \right)}$$

- If we are overshooting the target, reduce $w_j$ proportional to the value of $x_j$
- If we are undershooting the target, increase $w_j$ proportional to the value of $x_j$
- Otherwise, do nothing

# Overview

- Philosophical questions
- Derivatives: What are they good for?
- Linear regression
- Multiple linear regression
- Logistic regression

# Logistic Regression

- Linear regression: predict *y* given *x*
- Multiple linear regression: predict *y* given *x_1, x_2, …, x_n*

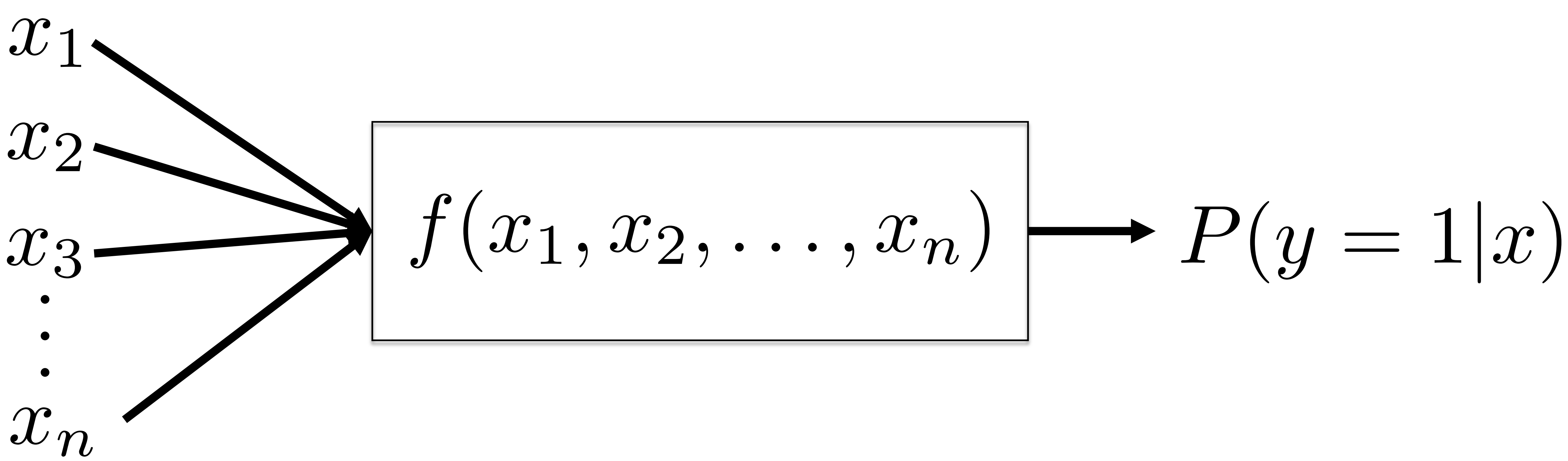
# Logistic Regression

- Logistic Regression: predict P(y=1|x_1, x_2, ..., x_n)
- We can use logistic regression to do binary classification.

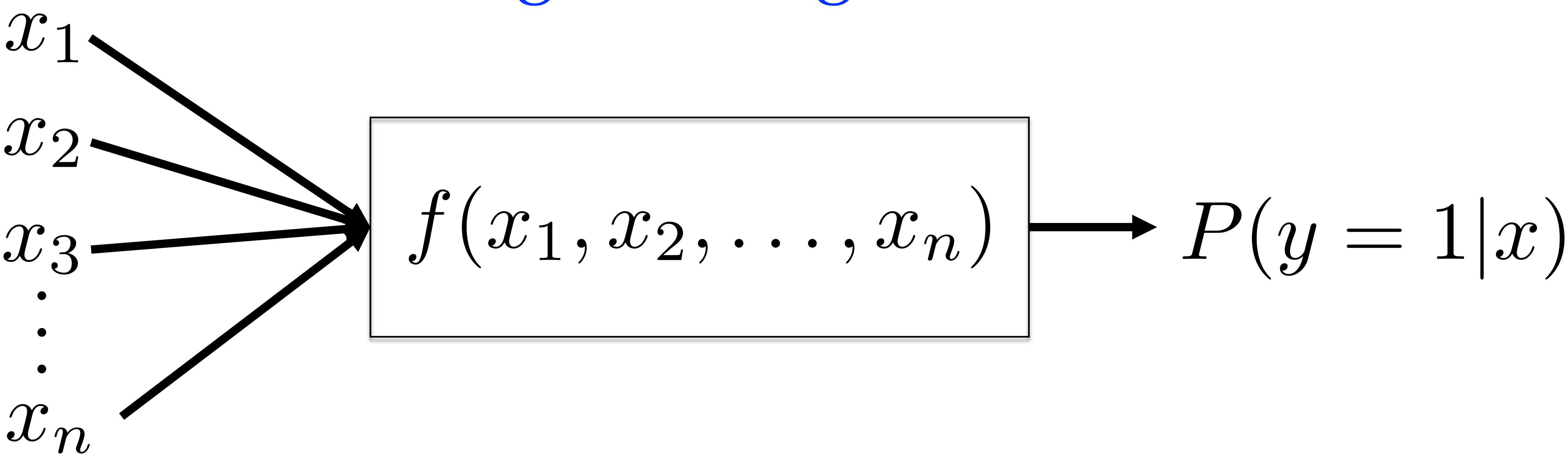
# Logistic Regression

| Size (feet) | No. of bedrooms | No. of floors | Age (years) | Price (x$1000) | Sell |
|---|---|---|---|---|---|
| 2,350 | 5 | 2 | 45 | 500 | 1 |
| 1,600 | 3 | 2 | 20 | 450 | 0 |
| 2,000 | 3 | 2 | 30 | 250 | 0 |
| 854 | 2 | 1 | 10 | 200 | 1 |
| 560 | 1 | 1 | 30 | 180 | 0 |

# Logistic Regression



# Logistic Regression

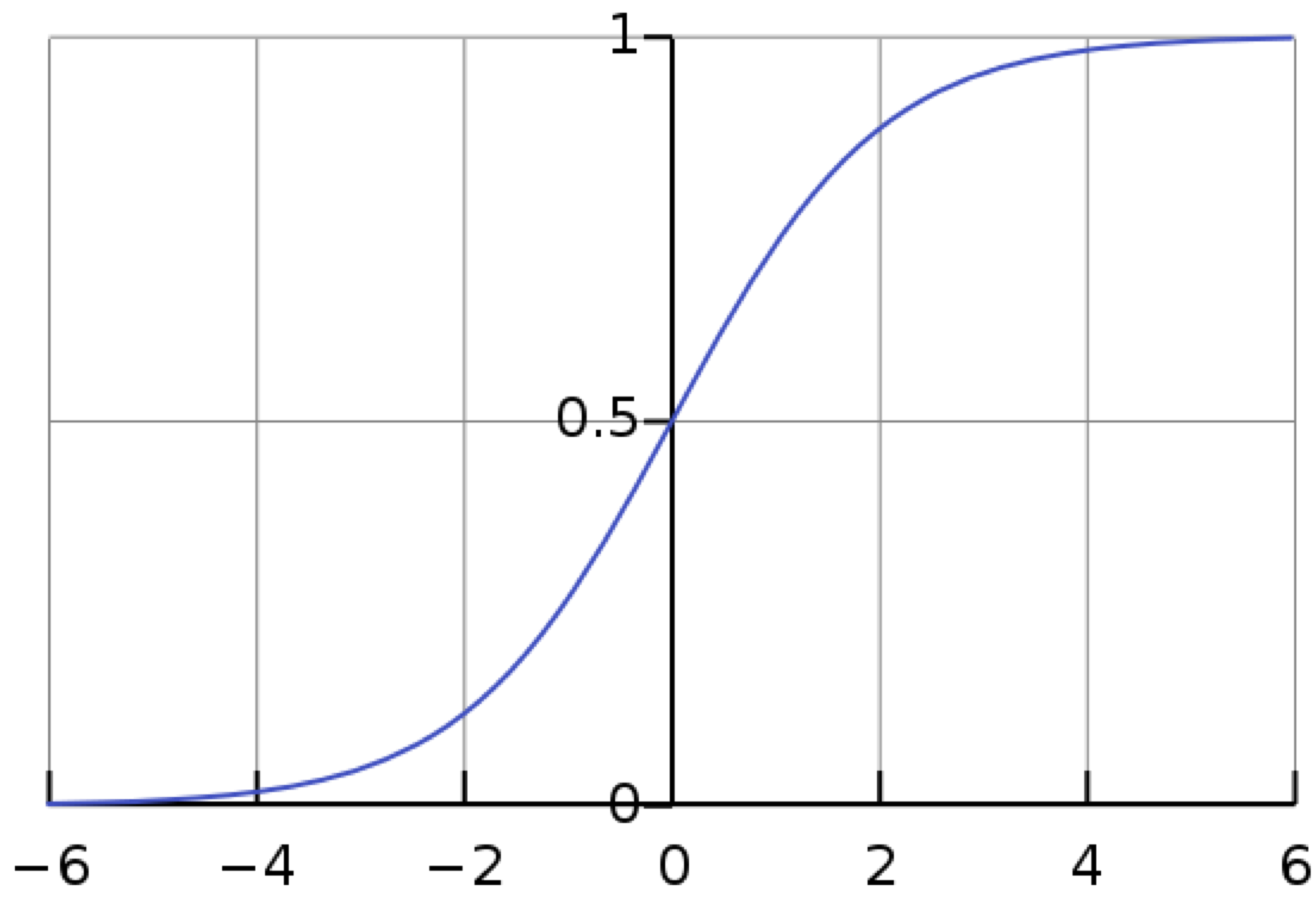



$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$z = \sum_{j=1}^{n} (w_j x_j) + b$$

# Logistic Regression

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



# Logistic Regression

- Loss Function: the discrepancy between the predicted and actual output values for a single training instance

$$z = \sum_{j=1}^{n} (w_j x_j) + b$$

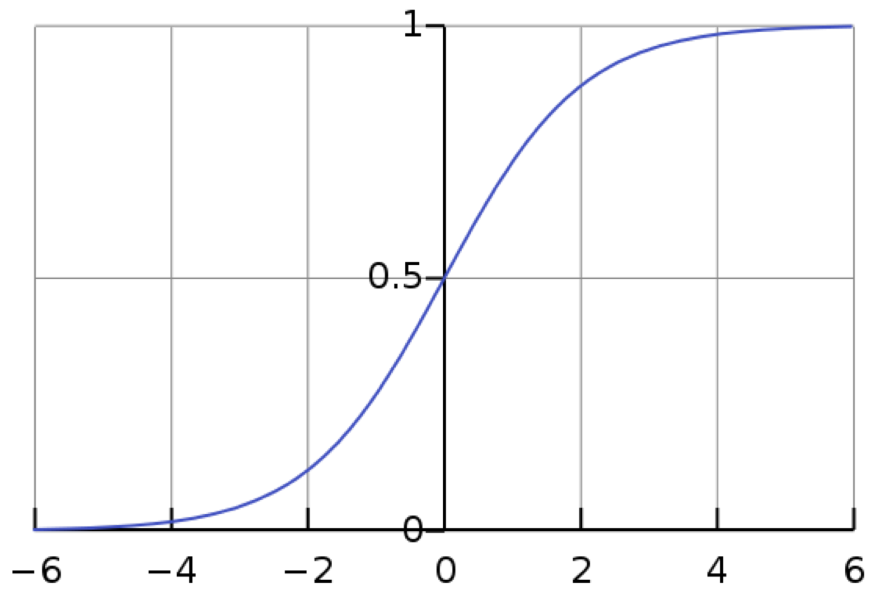
$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = -\left(y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})\right)$$

# Logistic Regression

- Loss Function: the discrepancy between the predicted and actual output values for <u>a single</u> training instance

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = -\left(y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})\right)$$

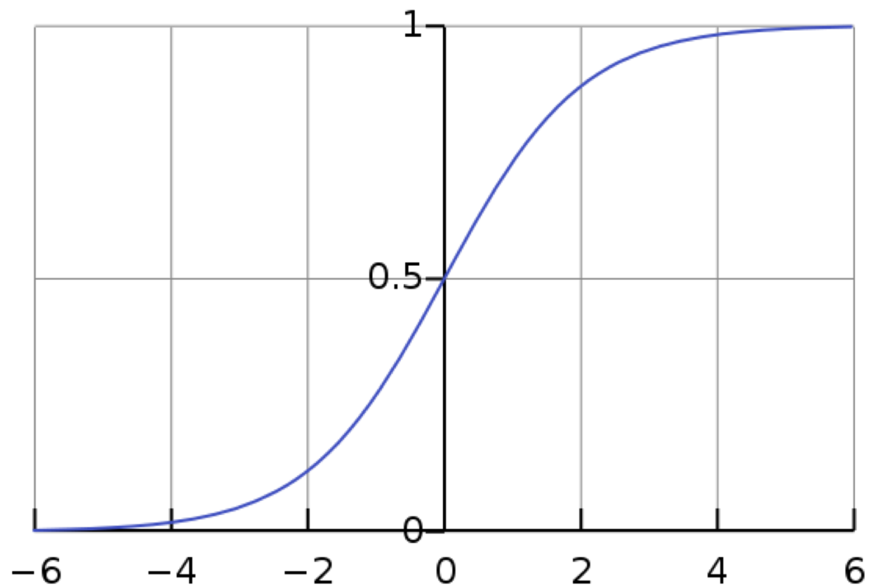



- If the true value is 1, we want the predicted value to be <u>high</u>.
- Remember: log(1) = 0

# Logistic Regression

- Loss Function: the discrepancy between the predicted and actual output values for <u>a single</u> training instance

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = -\left(y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})\right)$$




- If the true value is 0, we want the predicted value to be <u>low</u>.
- Remember: $\log(1) = 0$

# Logistic Regression

- Loss Function: the discrepancy between the predicted and actual output values for a single training instance

$$z = \sum_{j=1}^{n} (w_j x_j) + b$$

$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = -\Big(y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})\Big)$$

$$\frac{d}{db}\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = \hat{y}^{(i)} - y^{(i)}$$

# Logistic Regression

- Loss Function: the discrepancy between the predicted and actual output values for a single training instance

$$z = \sum_{j=1}^{n} (w_j x_j) + b$$

$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = -\Big(y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})\Big)$$

$$\frac{d}{dw_j} \mathcal{L}(y^{(i)}, \hat{y}^{(i)}) = (\hat{y}^{(i)} - y^{(i)}) x_j$$

# Logistic Regression:
## Gradient Descent

- Repeat many times (or until convergence):

$$b \leftarrow b - \alpha \frac{1}{m} \sum_{i=1}^{m} \left( \hat{y}^{(i)} - y^{(i)} \right)$$

$$w_j \leftarrow w_j - \alpha \frac{1}{m} \sum_{i=1}^{m} \left( (\hat{y}^{(i)} - y^{(i)}) x_j^{(i)} \right)$$

# Logistic Regression:
## Gradient Descent

- Repeat many times (or until convergence):

$$b \leftarrow b - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(\hat{y}^{(i)} - y^{(i)}\right)}$$

- If we are overshooting the target, reduce $b$
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- Otherwise, do nothing

# Logistic Regression:
## Gradient Descent

- Repeat many times (or until convergence):

$$w_j \leftarrow w_j - \alpha \boxed{\frac{1}{m} \sum_{i=1}^{m} \left( (\hat{y}^{(i)} - y^{(i)}) x_j^{(i)} \right)}$$

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# The Big Picture!

- Linear regression, multiple linear regression and logistic regression are examples of linear models
- Internally, linear models output a prediction based on a weighted combination of input features
- Features that are positively correlated with the target output get a positive weight
- Features that are negatively correlated with the target output get a negative weight
- Features that are uncorrelated with the target output get a zero weight