1: Vector Space Model (40%)

In the vector space model, the input query and the documents in the collection are represented as vectors in $\mathcal{V}$-dimensional space, where $\mathcal{V}$ denotes the size of the indexed vocabulary (i.e., the number of unique terms in the collection). Given a query, documents are scored (and ranked) based on their vector-space similarity to the query. In class, we talked about two vector-space similarity measures: (1) the *inner product* and (2) the *cosine similarity*. The goal of this question is to understand their differences.

Suppose we have a collection of 8 documents (denoted as $D_1 - D_8$ below). Answer the following questions. Assume a *binary* text representation—a vector's value for a particular dimension (i.e., a particular index term) equals 1 if the term appears *at least* once and 0 otherwise.

- D_1: jack and jill went up the hill

- D_2: to fetch a pail of water

- D_3: jack fell down and broke his crown

- D_4: and jill came tumbling after

- D_5: up jack got and home did trot

- D_6: as fast as he could caper

- D_7: to old dame dob who patched his nob

- D_8: with vinegar and brown paper

(a) Given a query-vector q and a document-vector d, the *inner product* (i.e, the score given to document d for query q) is given by,

$$\text{inner product}(q, d) = \sum_{i=1}^{\mathcal{V}} (q_i \times d_i)$$

Using the *inner product*, what is the score given to each document $D_1 - D_8$ in response to the query "jack"?

(b) Given a query-vector q and a document-vector d, the *cosine similarity* (i.e, the score given to document d for query q) is given by,

$$\text{cosine similarity}(q, d) = \frac{\sum_{i=1}^{\mathcal{V}} (q_i \times d_i)}{\sqrt{\sum_{i=1}^{\mathcal{V}} q_i^2} \times \sqrt{\sum_{i=1}^{\mathcal{V}} d_i^2}}$$

Using the *cosine similarity*, what is the score given to each document $D_1 - D_8$ in response to the query "jack"?

(c) For this particular query, scoring documents $D_1 - D_8$ using the inner-product and the cosine similarity would result in equal rankings (HINT: if they're not, you made a mistake). Why?

(d) Give an example of a query for which scoring documents $D_1 - D_8$ using the inner-product and the cosine similarity would result in *different* rankings. Explain your choice.

2: Term Weighting (30%)

The vector space model has the flexibility that it can accommodate different term-weighting schemes. Different term-weighting schemes make different assumptions about which terms are most important. Answer the following questions.

(a) According to a binary weighting scheme (1 if the term occurs, 0 if it doesn't), which are the most descriptive terms within a document?

(b) According to the TF (term-frequency) weighting scheme, which are the most descriptive terms within a document?

(c) According to the IDF (inverse-document frequency) weighting scheme, which are the most descriptive terms within a document?

(d) According to the TF.IDF (term-frequency multiplied by inverse document frequency) weighting scheme, which are the most descriptive terms within a document?

(e) Compute the TF.IDF weights for all seven terms in D_1. Use $D_1 - D_8$ to compute corpus statistics such as df_t. Do you notice anything strange? Why does this happen? Is it likely to happen in a more 'realistic' document collection?

3: Document Representation (30%)

Oftentimes, the documents we want to search have some amount of structure. Scholarly articles, for example, usually have a title, a set of authors, an abstract, a main body, a references section, and possibly an appendix. It turns out that weighting some parts of a document (e.g., the title) more heavily than other parts (e.g., the appendix) improves retrieval performance. The general idea is that a document with many of the query-terms appearing in the title should be scored and rank higher than a document with many of the query-terms appearing in the appendix—the title describes the main content of the document better than the appendix.

Suppose you have a collection of documents with two non-overlapping fields: a TITLE field and a BODY field. And, suppose you have access to an out-of-the-box search engine that performs vector space model retrieval using a binary text representation (1's and 0's) and inner-product scoring. Your goal is to design a solution that weights the TITLE field more than the BODY field. In other words, if you have a query with a single query term (e.g., "jack"), you want a document that has "jack" in the title (and nowhere else) to be scored and ranked higher than a document that has "jack" in the body (and nowhere else).

How would you do this? (HINT: there are many right answers. Be creative and have fun!).