

Naive Bayes Text Classification

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INLS 613: Text Data Mining

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Outline

Basic Probability and Notation

Bayes Law and Naive Bayes Classification

Smoothing

Class Prior Probabilities

Naive Bayes Classification

Summary

Crash Course in Basic Probability

Discrete Random Variable

- A is a discrete random variable if:
 - ▶ A describes an event with a finite number of possible outcomes (discrete vs continuous)
 - ▶ A describes an event whose outcomes have some degree of uncertainty (random vs. pre-determined)

Discrete Random Variables

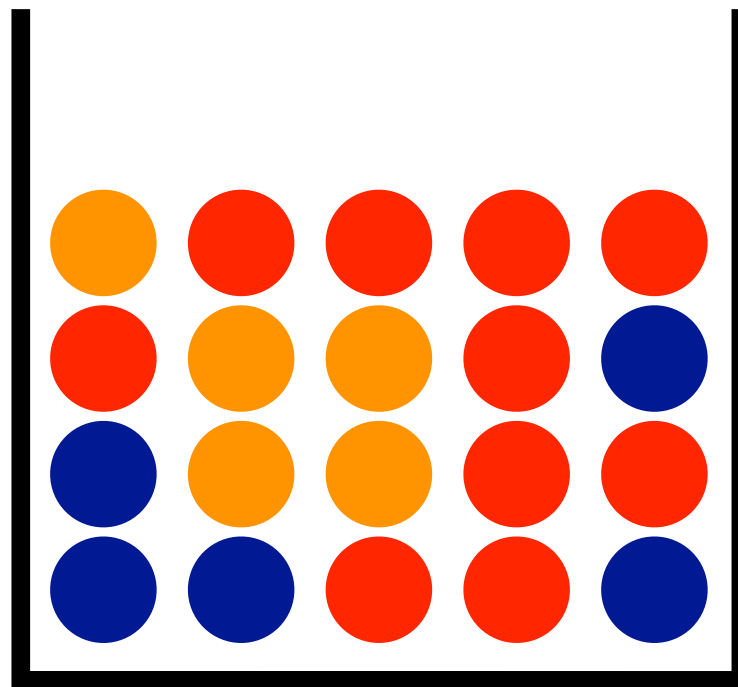
Examples

- A = the outcome of a coin-flip
 - ▶ outcomes: heads, tails
- A = it will rain tomorrow
 - ▶ outcomes: rain, no rain
- A = you have the flu
 - ▶ outcomes: flu, no flu
- A = your final grade in this class
 - ▶ outcomes: F, L, P, H

Discrete Random Variables

Examples

- A = the color of a ball pulled out from this bag
 - ▶ outcomes: RED, BLUE, ORANGE

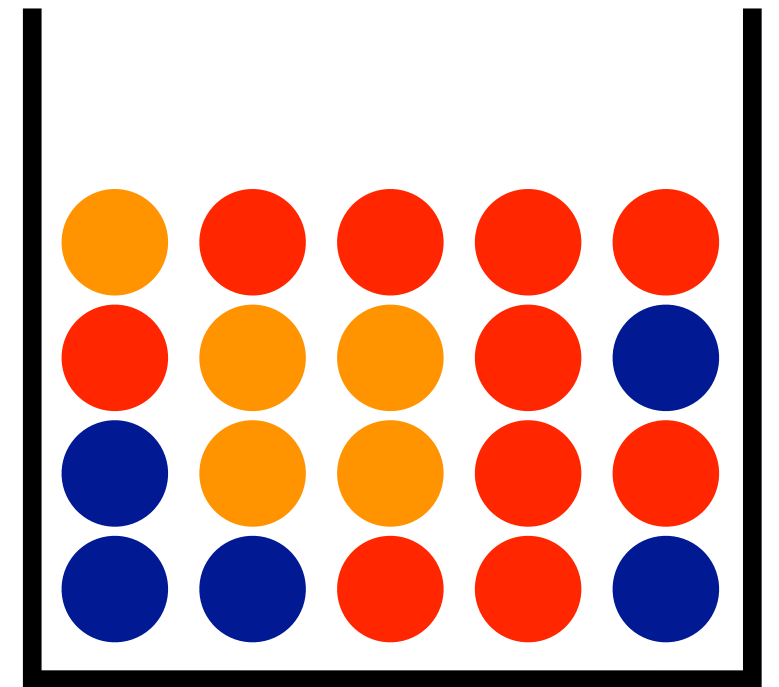


Probabilities

- Let $P(A=X)$ denote the probability that the outcome of event A equals X
- For simplicity, we often express $P(A=X)$ as $P(X)$
- Ex: $P(\text{RAIN})$, $P(\text{NO RAIN})$, $P(\text{FLU})$, $P(\text{NO FLU})$, ...

Probability Distribution

- A **probability distribution** gives the probability of each possible outcome of a random variable
- $P(\text{RED})$ = probability of pulling out a **red** ball
- $P(\text{BLUE})$ = probability of pulling out a **blue** ball
- $P(\text{ORANGE})$ = probability of pulling out an **orange** ball



Probability Distribution

- For it to be a probability distribution, two conditions must be satisfied:
 - ▶ the probability assigned to each possible outcome must be between 0 and 1 (inclusive)
 - ▶ the sum of probabilities assigned to all outcomes must equal 1

$$0 \leq P(\text{RED}) \leq 1$$

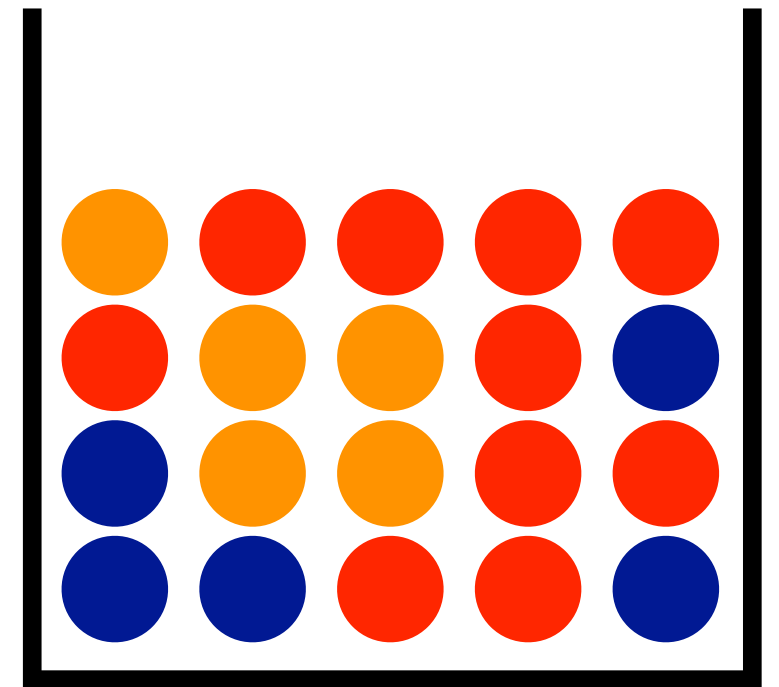
$$0 \leq P(\text{BLUE}) \leq 1$$

$$0 \leq P(\text{ORANGE}) \leq 1$$

$$P(\text{RED}) + P(\text{BLUE}) + P(\text{ORANGE}) = 1$$

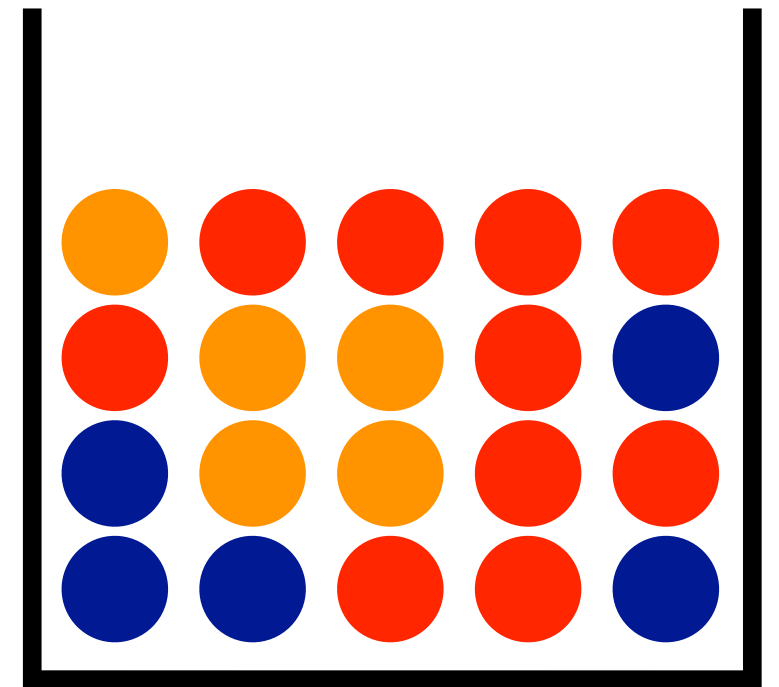
Probability Distribution Estimation

- Let's estimate these probabilities based on what we know about the contents of the bag
- $P(\text{RED}) = ?$
- $P(\text{BLUE}) = ?$
- $P(\text{ORANGE}) = ?$



Probability Distribution estimation

- Let's estimate these probabilities based on what we know about the contents of the bag
- $P(\text{RED}) = 10/20 = 0.5$
- $P(\text{BLUE}) = 5/20 = 0.25$
- $P(\text{ORANGE}) = 5/20 = 0.25$
- $P(\text{RED}) + P(\text{BLUE}) + P(\text{ORANGE}) = 1.0$



Probability Distribution

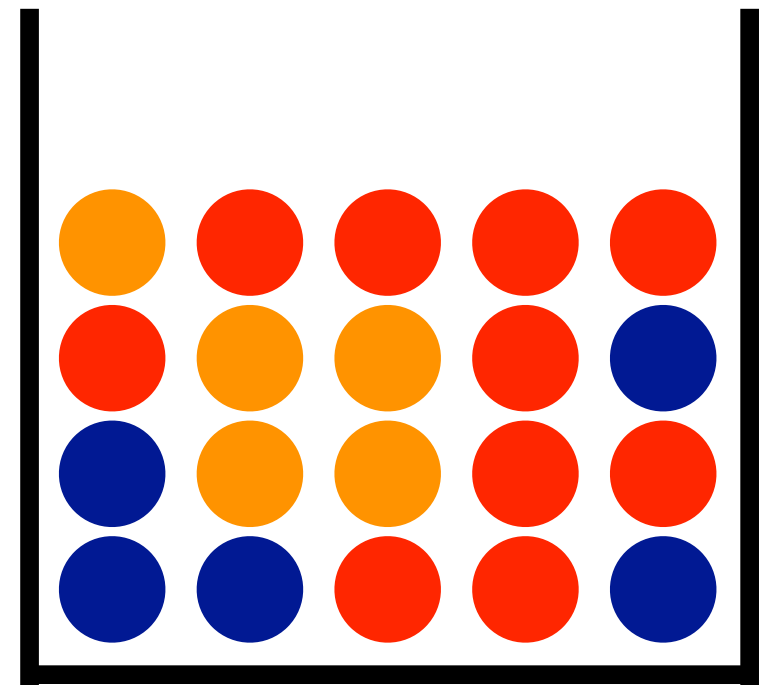
assigning probabilities to outcomes

- Given a probability distribution, we can assign probabilities to different outcomes
- I reach into the bag and pull out an **orange** ball. What is the probability of that happening?
- I reach into the bag and pull out two balls: one **red**, one **blue**. What is the probability of that happening?
- What about three **orange** balls?

$$P(\text{RED}) = 0.5$$

$$P(\text{BLUE}) = 0.25$$

$$P(\text{ORANGE}) = 0.25$$



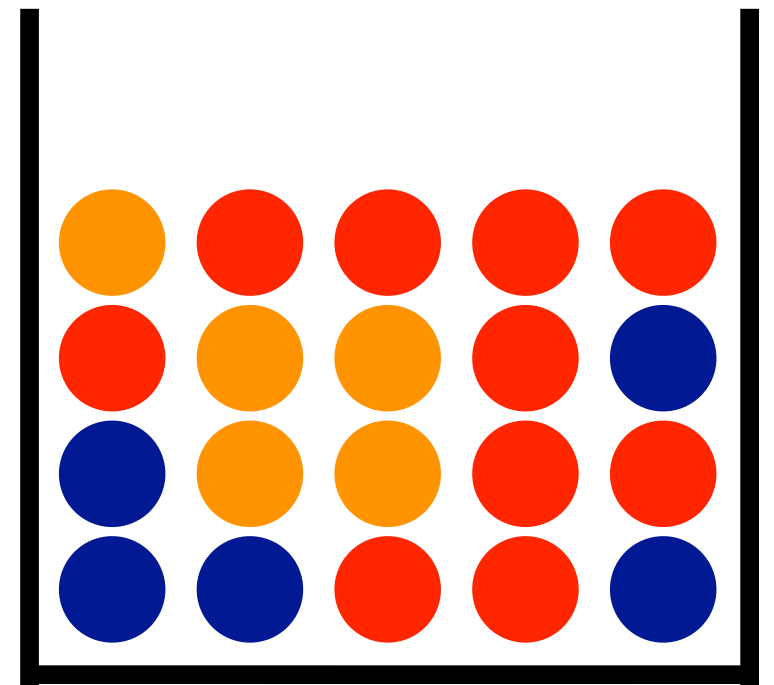
What can we do with a probability distribution?

- If we assume that each outcome is independent of previous outcomes, then the probability of a sequence of outcomes is calculated by multiplying the individual probabilities
- **Note:** we're assuming that when you take out a ball, you put it back in the bag before taking another

$$P(\text{RED}) = 0.5$$

$$P(\text{BLUE}) = 0.25$$

$$P(\text{ORANGE}) = 0.25$$



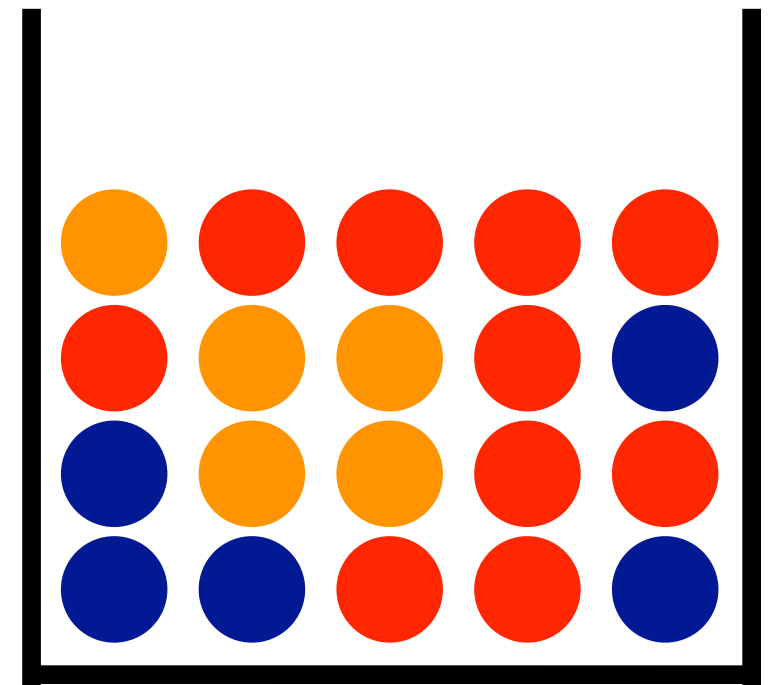
What can we do with a probability distribution?

- $P(\text{●}) = ??$
- $P(\text{●}) = ??$
- $P(\text{●} \text{ ●} \text{ ●}) = ??$
- $P(\text{●} \text{ ●} \text{ ●}) = ??$
- $P(\text{●} \text{ ●} \text{ ●}) = ??$
- $P(\text{●} \text{ ●} \text{ ●} \text{ ●}) = ??$

$$P(\text{RED}) = 0.5$$

$$P(\text{BLUE}) = 0.25$$

$$P(\text{ORANGE}) = 0.25$$



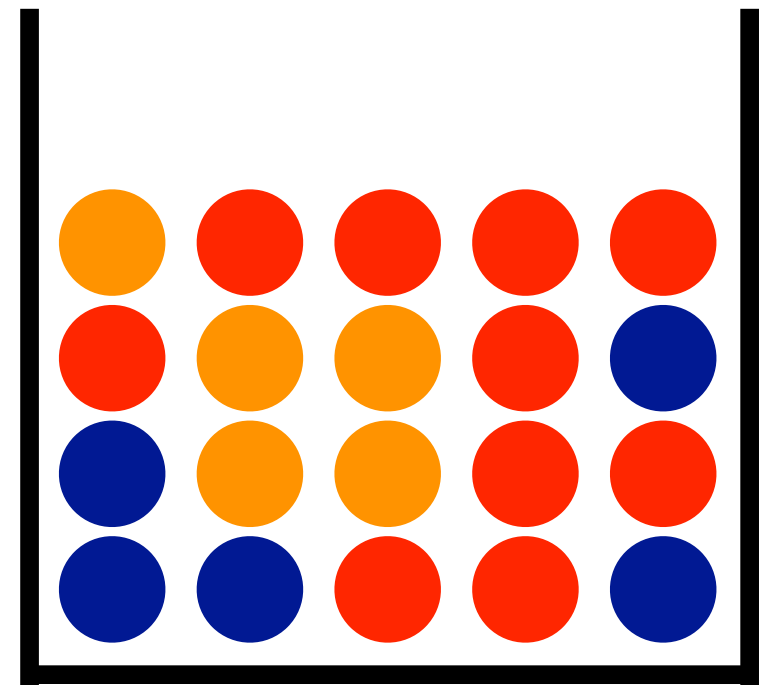
What can we do with a probability distribution?

- $P(\text{●}) = 0.25$
- $P(\text{●}) = 0.5$
- $P(\text{●} \text{ ●} \text{ ●}) = 0.25 \times 0.25 \times 0.25$
- $P(\text{●} \text{ ●} \text{ ●}) = 0.25 \times 0.25 \times 0.25$
- $P(\text{●} \text{ ●} \text{ ●}) = 0.25 \times 0.50 \times 0.25$
- $P(\text{●} \text{ ●} \text{ ●} \text{ ●}) = 0.25 \times 0.50 \times 0.25 \times 0.50$

$$P(\text{RED}) = 0.5$$

$$P(\text{BLUE}) = 0.25$$

$$P(\text{ORANGE}) = 0.25$$



Conditional Probability

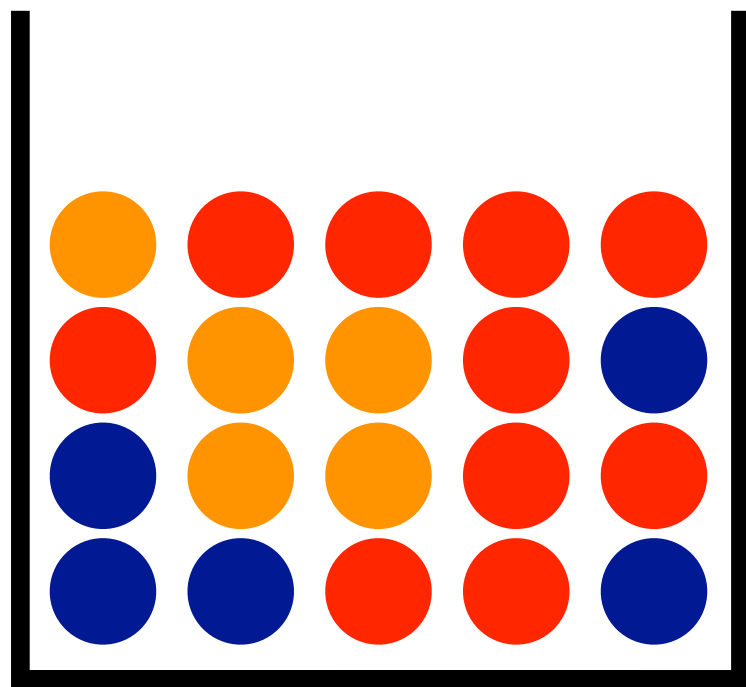
- $P(A,B)$: the probability that event A and event B both occur
- $P(A|B)$: the probability of event A occurring given prior knowledge that event B occurred

Conditional Probability

$$P(\text{RED}) = 0.50$$

$$P(\text{BLUE}) = 0.25$$

$$P(\text{ORANGE}) = 0.25$$



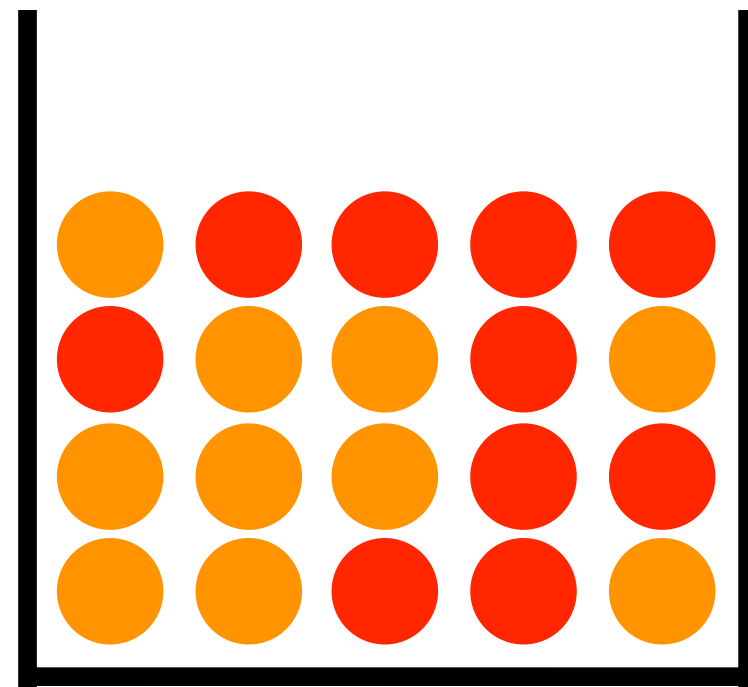
A

- $P(\text{BLUE} \mid A) = ??$
- $P(\text{RED} \mid A) = ??$
- $P(\text{ORANGE} \mid A) = ??$

$$P(\text{RED}) = 0.50$$

$$P(\text{BLUE}) = 0.00$$

$$P(\text{ORANGE}) = 0.50$$



B

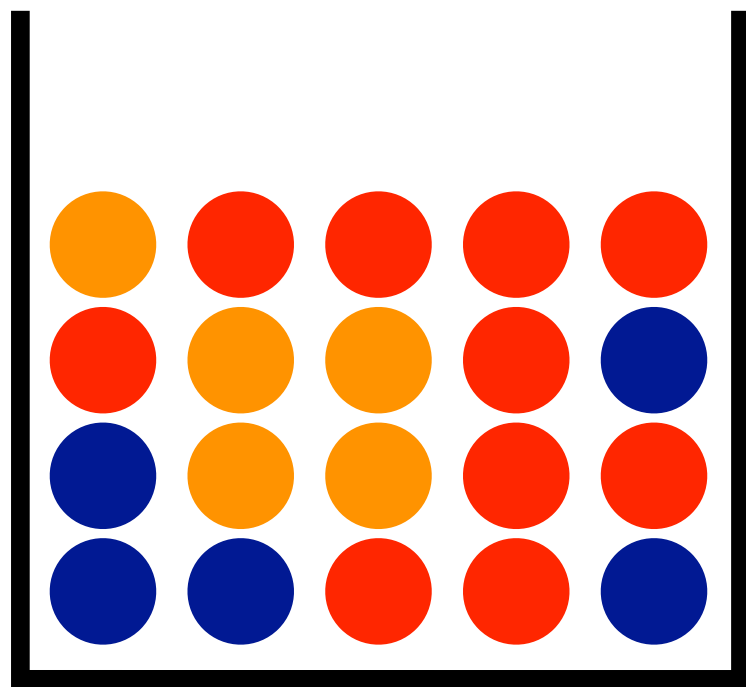
- $P(\text{BLUE} \mid B) = ??$
- $P(\text{RED} \mid B) = ??$
- $P(\text{ORANGE} \mid B) = ??$

Conditional Probability

$$P(\text{RED}) = 0.50$$

$$P(\text{BLUE}) = 0.25$$

$$P(\text{ORANGE}) = 0.25$$



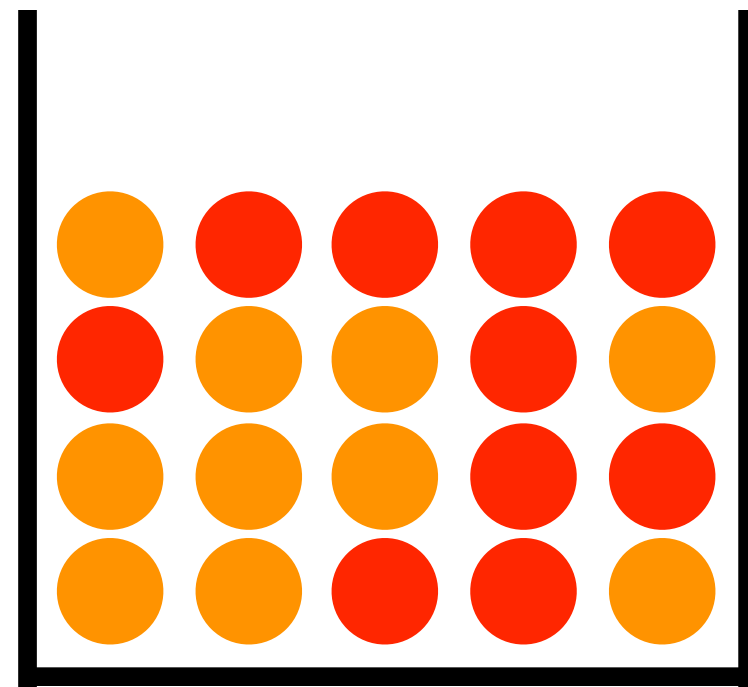
A

- $P(\text{BLUE} \mid A) = 0.25$
- $P(\text{RED} \mid A) = 0.50$
- $P(\text{ORANGE ORANGE ORANGE} \mid A) = 0.016$

$$P(\text{RED}) = 0.50$$

$$P(\text{BLUE}) = 0.00$$

$$P(\text{ORANGE}) = 0.50$$



B

- $P(\text{BLUE} \mid B) = 0.00$
- $P(\text{RED ORANGE} \mid B) = 0.25$
- $P(\text{ORANGE RED BLUE} \mid B) = 0.00$

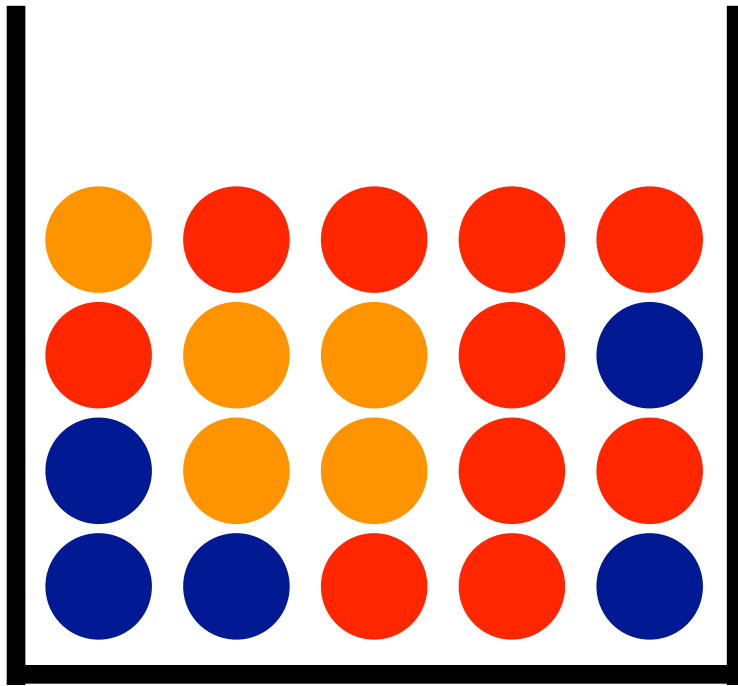
Chain Rule

- $P(A, B) = P(A|B) \times P(B)$
- Example:
 - ▶ probability that it will rain today (**B**) and tomorrow (**A**)
 - ▶ probability that it will rain today (**B**)
 - ▶ probability that it will rain tomorrow (**A**) given that it will rain today (**B**)

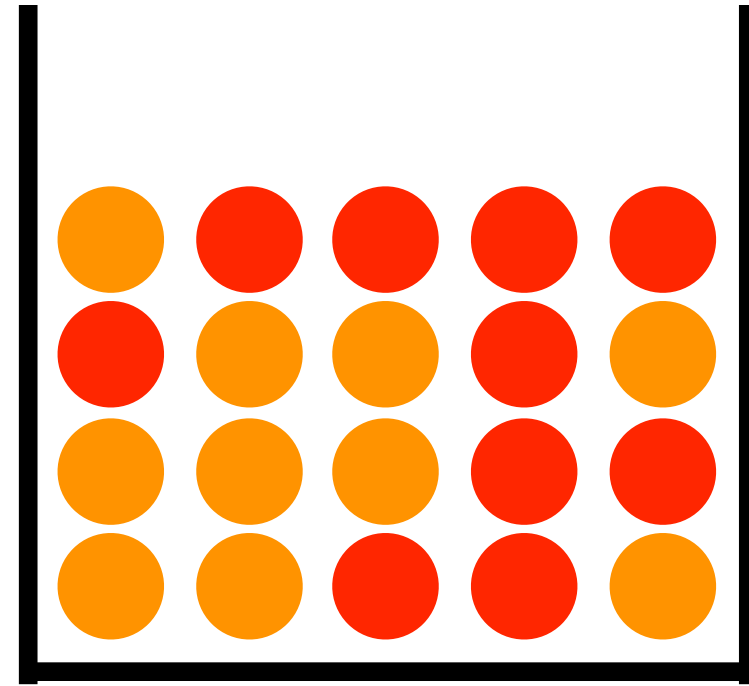
Independence

- $P(A, B) = P(A|B) \times P(B) = P(A) \times P(B)$
- Example:
 - ▶ probability that it will rain today (B) and tomorrow (A)
 - ▶ probability that it will rain today (B)
 - ▶ probability that it will rain tomorrow (A) given that it will rain today (B)
 - ▶ probability that it will rain tomorrow (A)

Independence



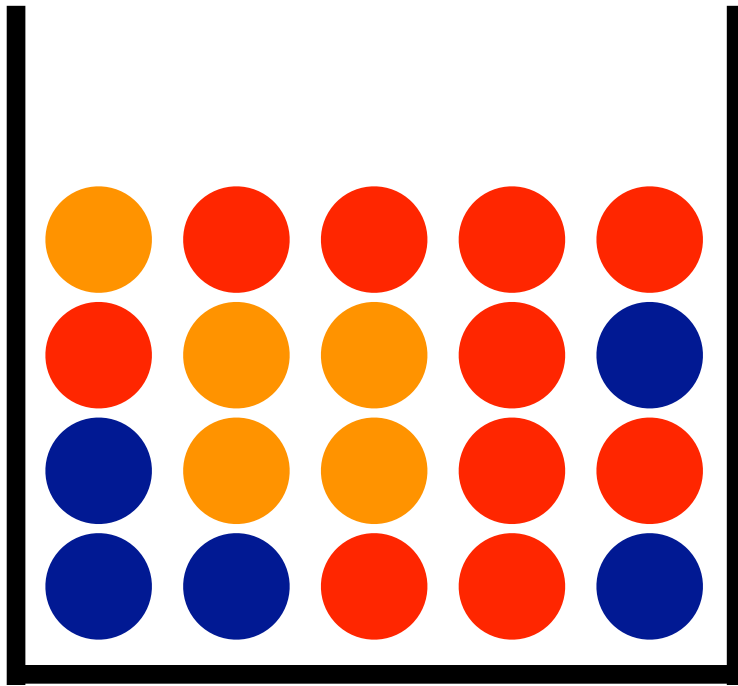
A



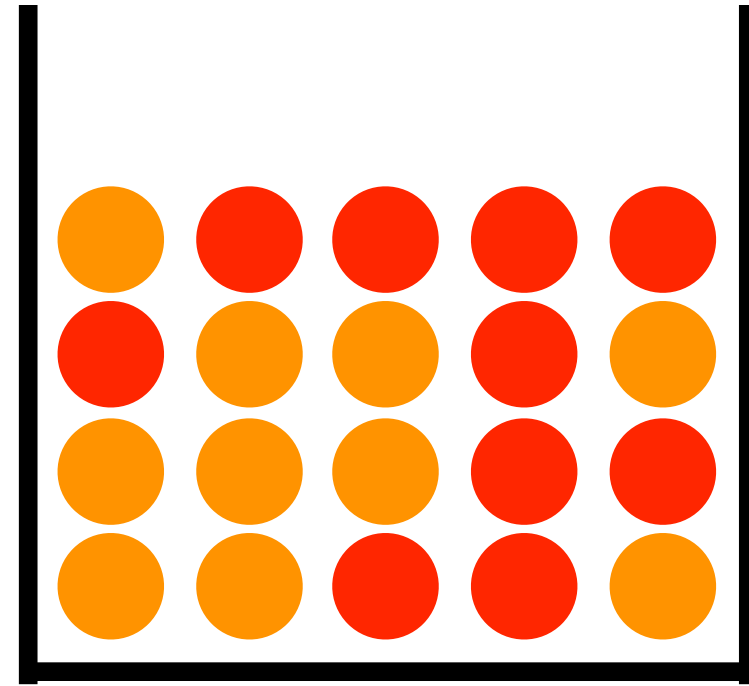
B

$$P(\text{Blue} \mid A) \neq P(\text{Blue})$$

Independence



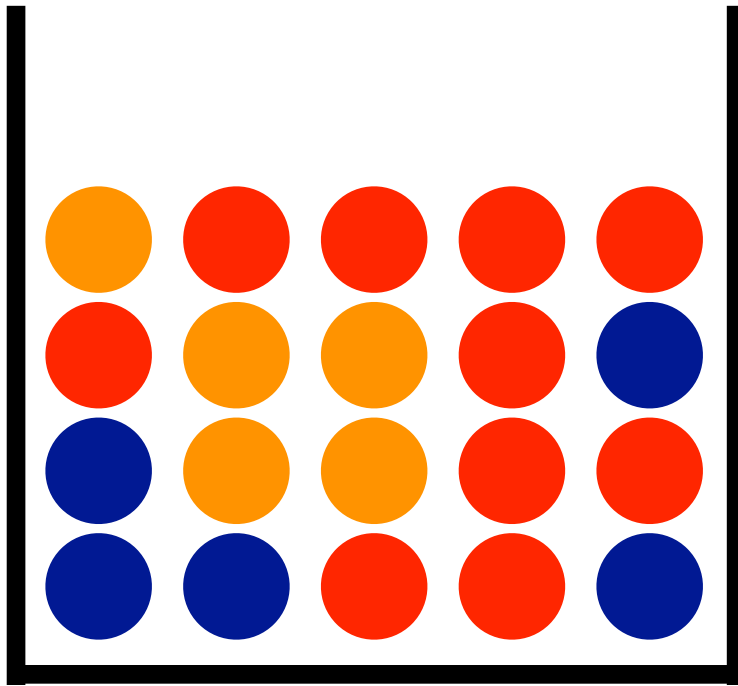
A



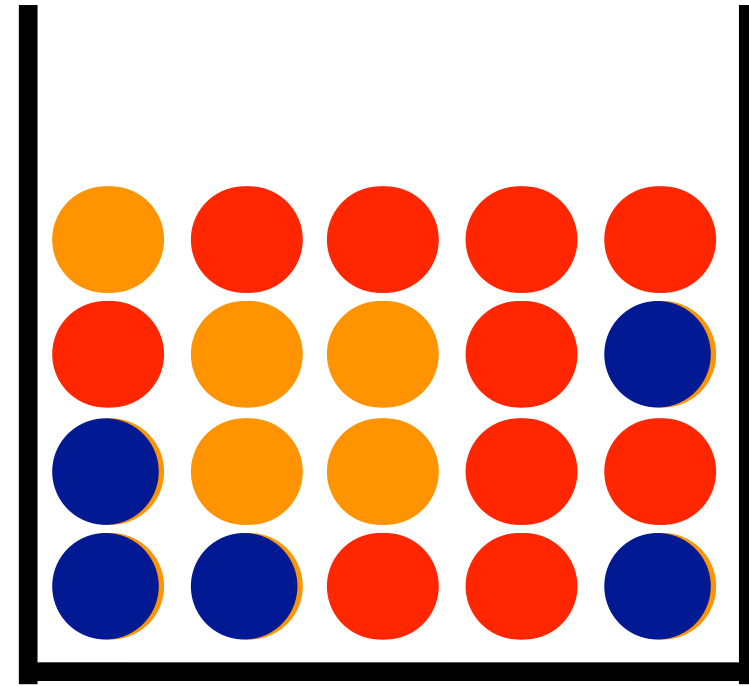
B

$$P(\text{Blue} \mid A) > P(\text{Blue})$$

Independence



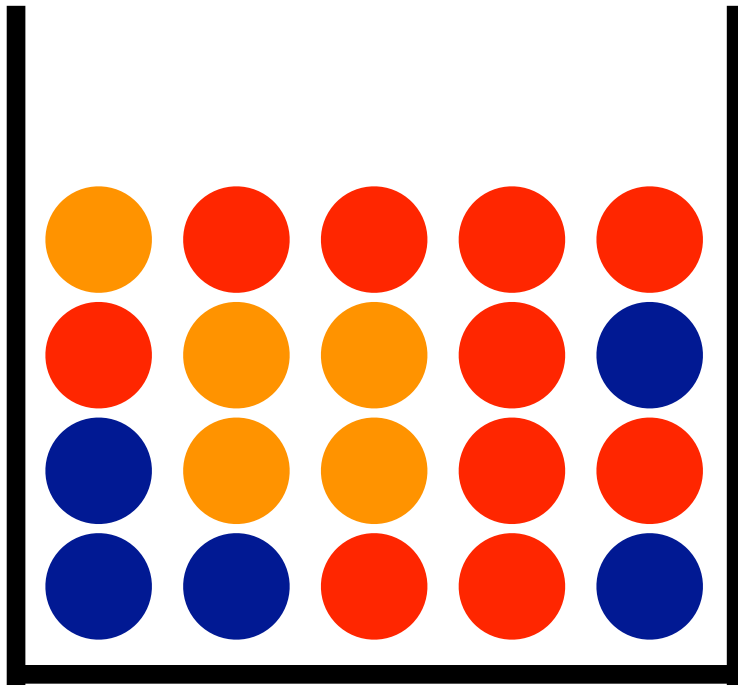
A



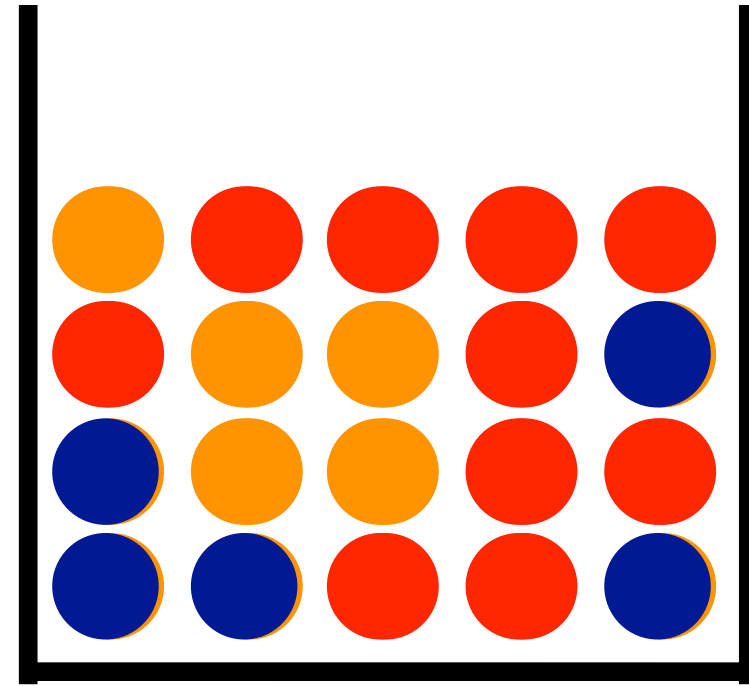
B

$$P(\text{Blue} | A) \neq P(\text{Blue})$$

Independence



A



B

$$P(\bullet | A) = P(\bullet)$$

Outline

Basic Probability and Notation

Bayes Law and Naive Bayes Classification

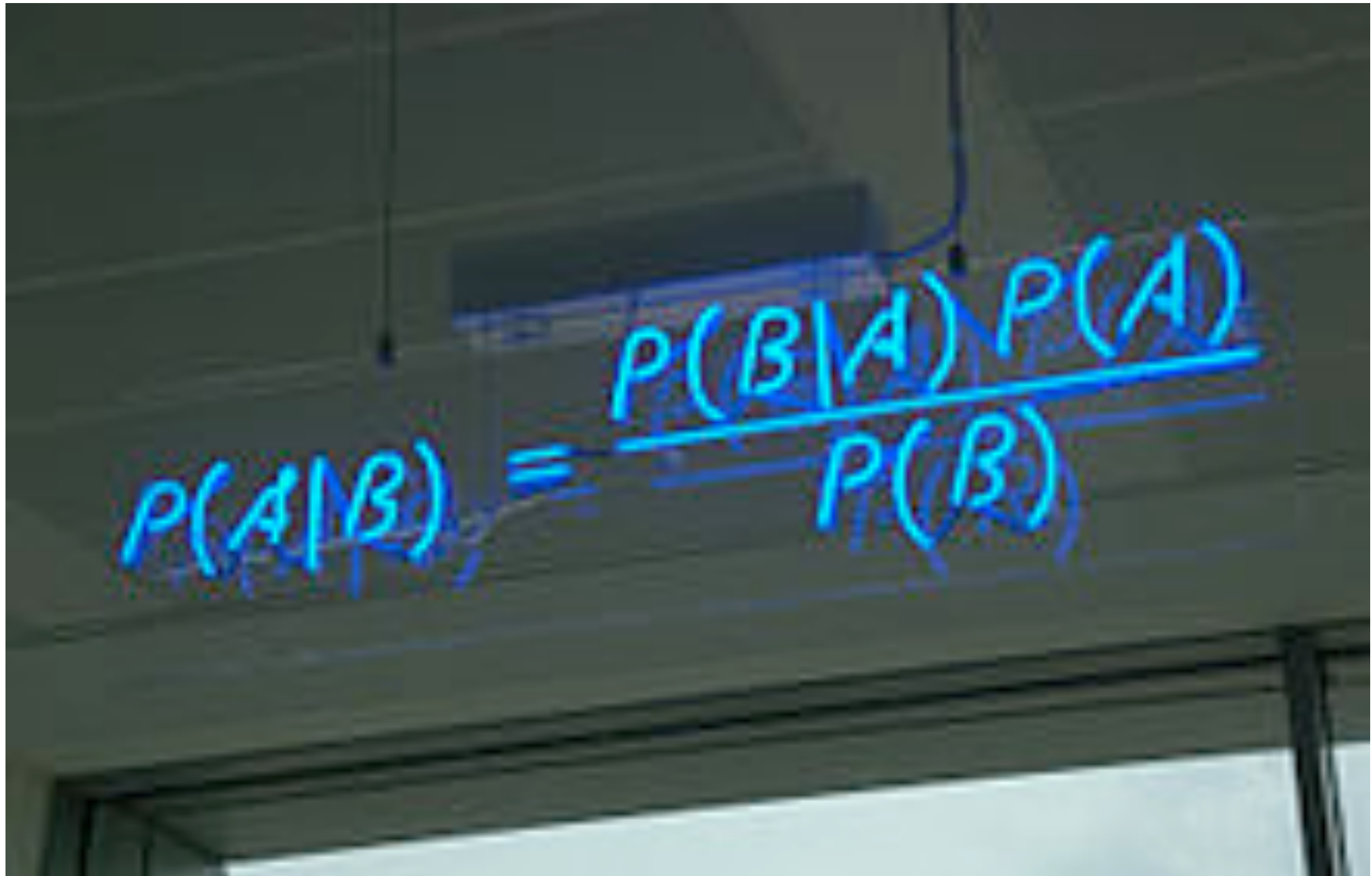
Smoothing

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Naive Bayes Classification

Summary

Bayes' Law



A photograph of a chalkboard with the formula for Bayes' Law written in white chalk. The formula is $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$. The chalkboard is dark, and the lighting is somewhat dim, with some visible chalk dust or smudges. The formula is written in a clear, handwritten style.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes' Law

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Derivation of Bayes' Law

$$P(A, B) = P(A, B)$$

Always true!

$$P(A|B) \times P(B) = P(B|A) \times P(A)$$

Chain Rule!

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Divide both
sides by P(B)!

Naive Bayes Classification

example: positive/negative movie reviews

Bayes Rule

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Confidence of
POS prediction
given instance D

$$P(POS|D) = \frac{P(D|POS) \times P(POS)}{P(D)}$$

Confidence of
NEG prediction
given instance D

$$P(NEG|D) = \frac{P(D|NEG) \times P(NEG)}{P(D)}$$

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D , predict positive (**POS**) if:

$$P(POS|D) \geq P(NEG|D)$$

- Otherwise, predict negative (**NEG**)

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D , predict positive (**POS**) if:

$$\frac{P(D|POS) \times P(POS)}{P(D)} \geq \frac{P(D|NEG) \times P(NEG)}{P(D)}$$

- Otherwise, predict negative (**NEG**)

Naive Bayes Classification

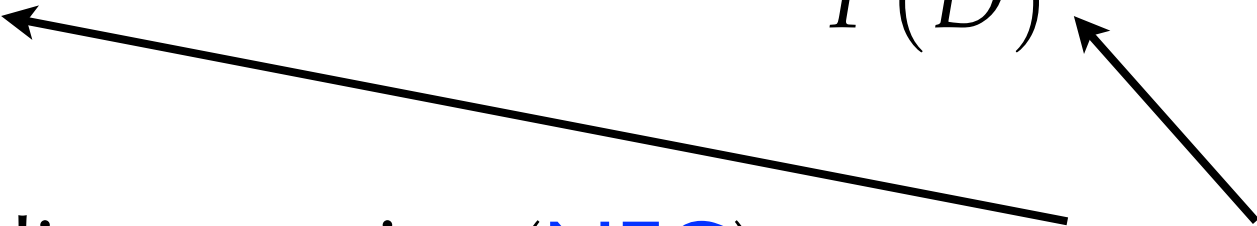
example: positive/negative movie reviews

- Given instance D , predict positive (**POS**) if:

$$\frac{P(D|POS) \times P(POS)}{P(D)} \geq \frac{P(D|NEG) \times P(NEG)}{P(D)}$$

- Otherwise, predict negative (**NEG**)

Are these
necessary?



Naive Bayes Classification

example: positive/negative movie reviews

- Given instance D , predict positive (**POS**) if:

$$P(D|POS) \times P(POS) \geq P(D|NEG) \times P(NEG)$$


- Otherwise, predict negative (**NEG**)

Naive Bayes Classification

example: positive/negative movie reviews

- Our next goal is to estimate these parameters from the training data!

- $P(\text{NEG}) = ??$



Easy!

- $P(\text{POS}) = ??$

- $P(D|\text{NEG}) = ??$

- $P(D|\text{POS}) = ??$



Not so easy!

Naive Bayes Classification

example: positive/negative movie reviews

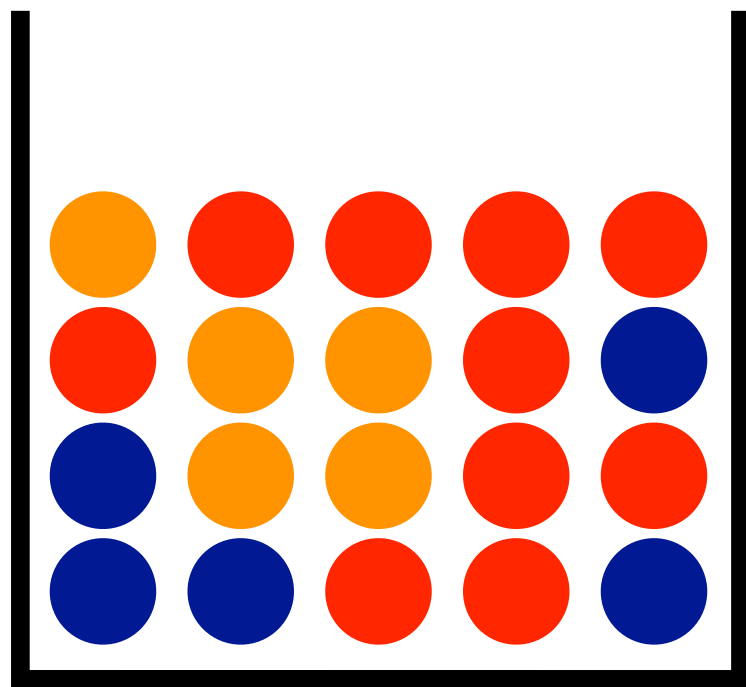
- Our next goal is to estimate these parameters from the training data!
- $P(\text{NEG})$ = % of training set documents that are **NEG**
- $P(\text{POS})$ = % of training set documents that are **POS**
- $P(D|\text{NEG})$ = ??
- $P(D|\text{POS})$ = ??

Remember Conditional Probability?

$$P(\text{RED}) = 0.50$$

$$P(\text{BLUE}) = 0.25$$

$$P(\text{ORANGE}) = 0.25$$



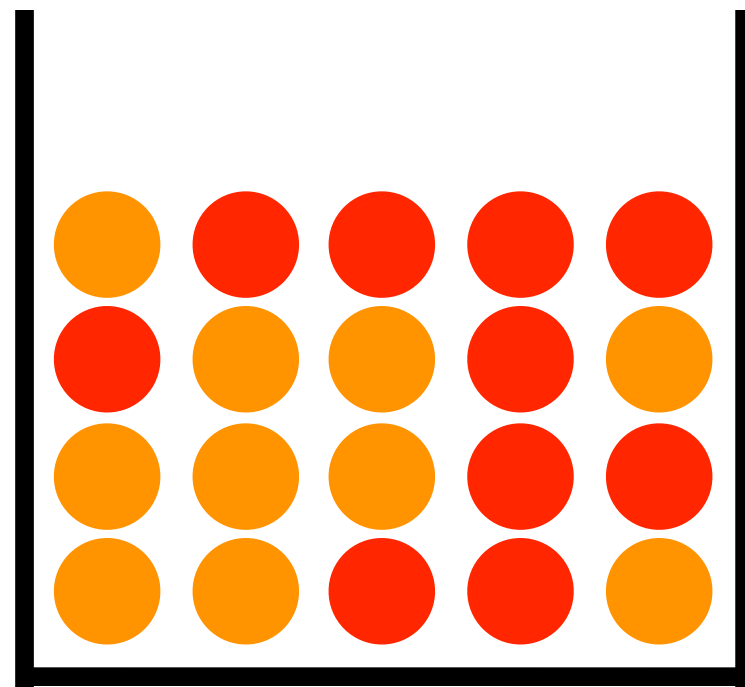
A

- $P(\text{BLUE} \mid A) = 0.25$
- $P(\text{RED} \mid A) = 0.50$
- $P(\text{ORANGE} \mid A) = 0.25$

$$P(\text{RED}) = 0.50$$

$$P(\text{BLUE}) = 0.00$$

$$P(\text{ORANGE}) = 0.50$$

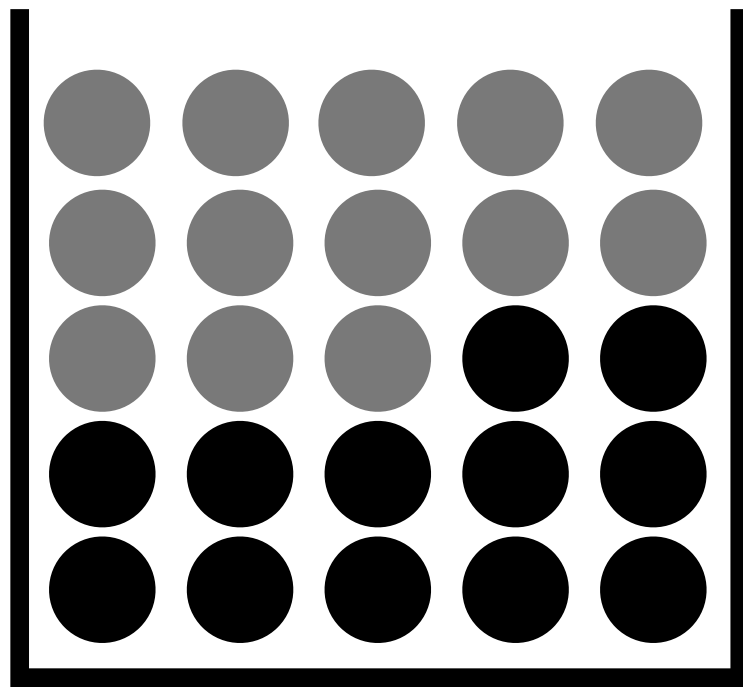


B

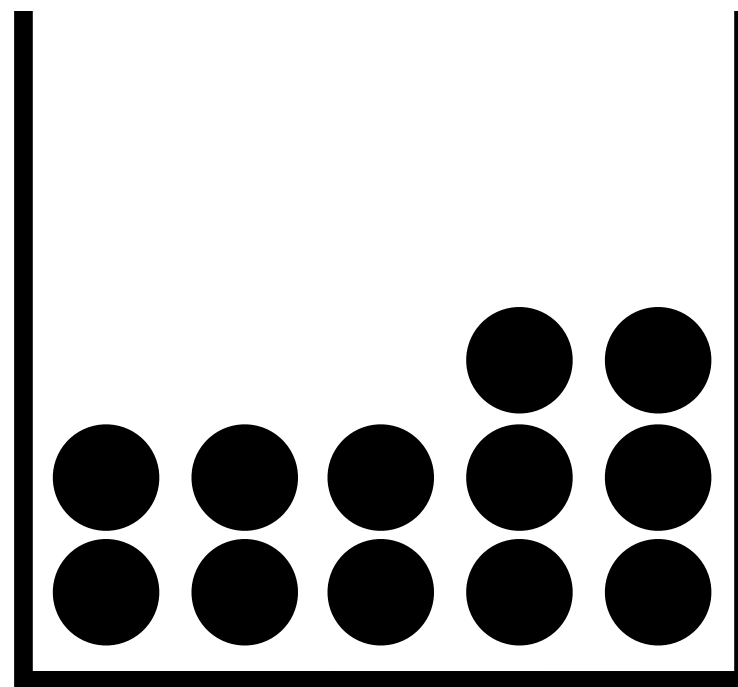
- $P(\text{BLUE} \mid B) = 0.00$
- $P(\text{RED} \mid B) = 0.50$
- $P(\text{ORANGE} \mid B) = 0.50$

Naive Bayes Classification

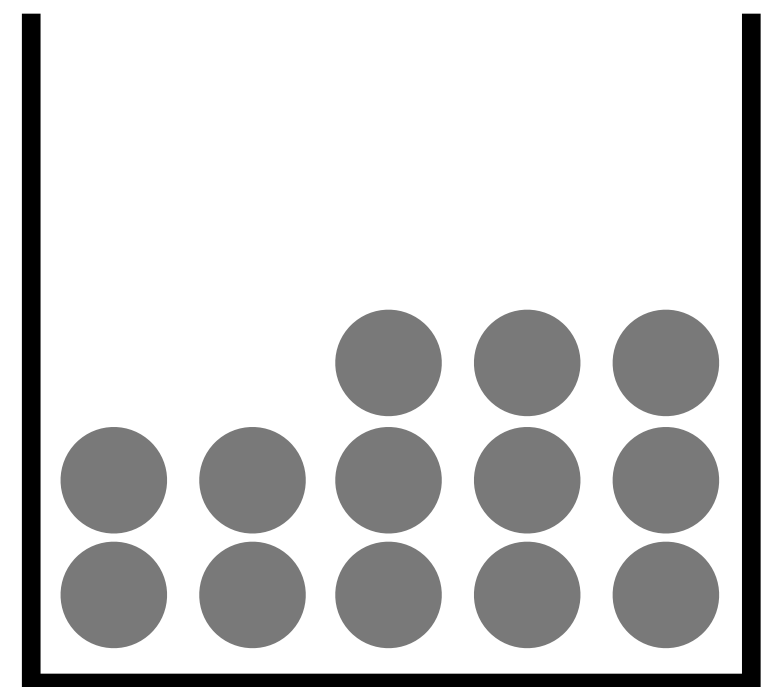
example: positive/negative movie reviews



Training
Instances



Positive Training
Instances



Negative Training
Instances

$$P(D|POS) = ??$$

$$P(D|NEG) = ??$$

Naive Bayes Classification

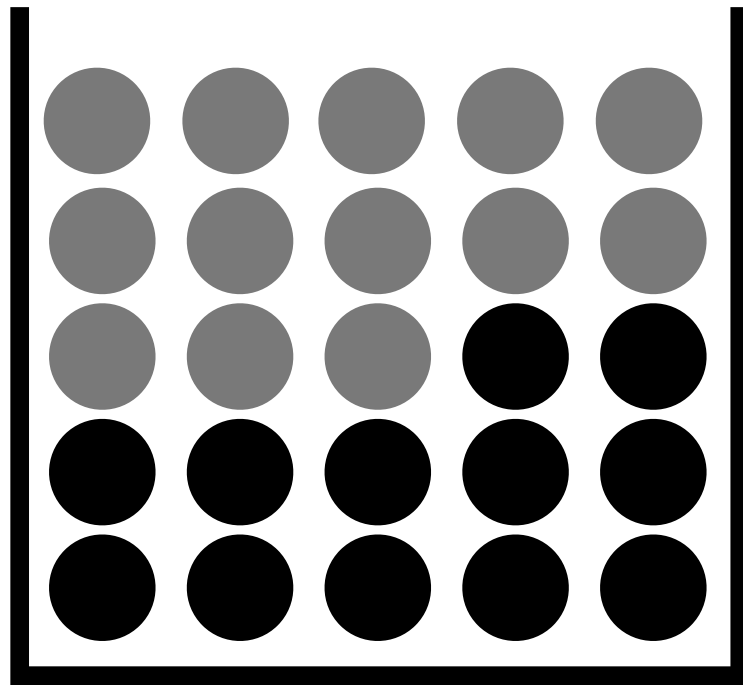
example: positive/negative movie reviews

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	...	w_n	sentiment
1	0	1	0	1	0	0	1	...	0	positive
0	1	0	1	1	0	1	1	...	0	positive
0	1	0	1	1	0	1	0	...	0	positive
0	0	1	0	1	1	0	1	...	1	positive
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	...	⋮	⋮
1	1	0	1	1	0	0	1	...	1	positive

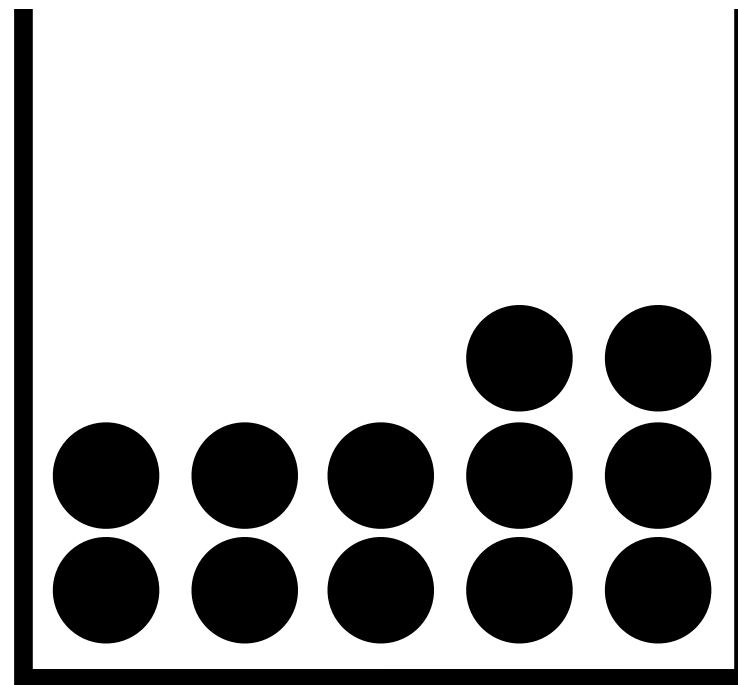
Naive Bayes Classification

example: positive/negative movie reviews

- We have a problem! What is it?

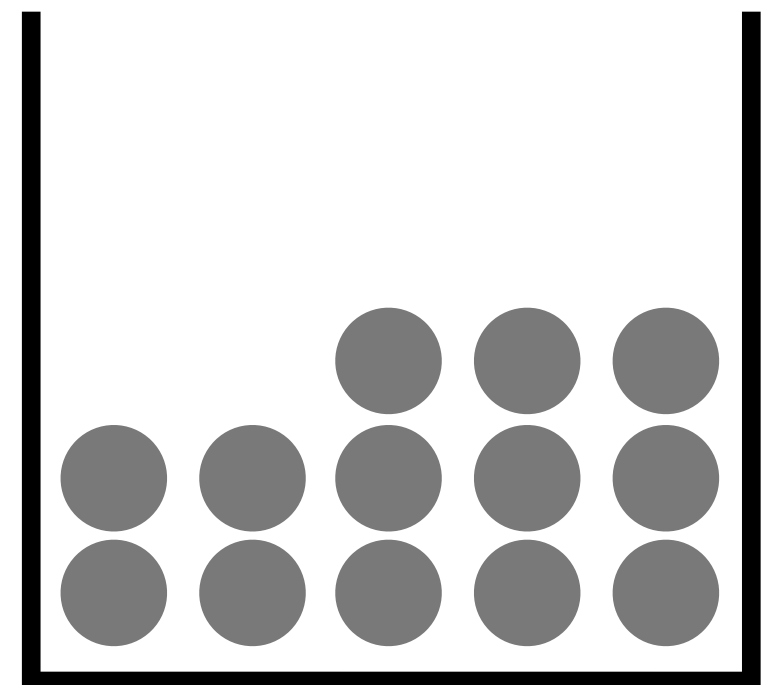


Training
Instances



Positive Training
Instances

$$P(D|POS) = ??$$



Negative Training
Instances

$$P(D|NEG) = ??$$

Naive Bayes Classification

example: positive/negative movie reviews

- We have a problem! What is it?
- Assuming n binary features, the number of possible combinations is 2^n
- $2^{1000} = 1.071509e+301$
- And in order to estimate the probability of each combination, we would require multiple occurrences of each combination in the training data!
- We could never have enough training data to reliably estimate $P(D|NEG)$ or $P(D|POS)$!

Naive Bayes Classification

example: positive/negative movie reviews

- **Assumption:** given a particular class value (i.e, **POS** or **NEG**), the value of a particular feature is independent of the value of other features
- In other words, the value of a particular feature is only dependent on the class value

Naive Bayes Classification

example: positive/negative movie reviews

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	...	w_n	sentiment
1	0	1	0	1	0	0	1	...	0	positive
0	1	0	1	1	0	1	1	...	0	positive
0	1	0	1	1	0	1	0	...	0	positive
0	0	1	0	1	1	0	1	...	1	positive
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	...	⋮	⋮
1	1	0	1	1	0	0	1	...	1	positive

Naive Bayes Classification

example: positive/negative movie reviews

- **Assumption:** given a particular class value (i.e, **POS** or **NEG**), the value of a particular feature is independent of the value of other features

- **Example:** we have seven features and **D = 1 0 1 1 0 1 1**

- $P(1 0 1 1 0 1 1 | \text{POS}) =$

$$P(w_1=1 | \text{POS}) \times P(w_2=0 | \text{POS}) \times P(w_3=1 | \text{POS}) \times P(w_4=1 | \text{POS}) \times P(w_5=0 | \text{POS}) \times P(w_6=1 | \text{POS}) \times P(w_7=1 | \text{POS})$$

- $P(1 0 1 1 0 1 1 | \text{NEG}) =$

$$P(w_1=1 | \text{NEG}) \times P(w_2=0 | \text{NEG}) \times P(w_3=1 | \text{NEG}) \times P(w_4=1 | \text{NEG}) \times P(w_5=0 | \text{NEG}) \times P(w_6=1 | \text{NEG}) \times P(w_7=1 | \text{NEG})$$

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_i = \text{!} | \text{POS})$?

Naive Bayes Classification

example: positive/negative movie reviews

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	...	w_n	sentiment
1	0	1	0	1	0	0	1	...	0	positive
0	1	0	1	1	0	1	1	...	0	negative
0	1	0	1	1	0	1	0	...	0	negative
0	0	1	0	1	1	0	1	...	1	positive
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	...	⋮	⋮
1	1	0	1	1	0	0	1	...	1	negative

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_i = \text{!} | \text{POS})$?

	POS	NEG	
$w_i = \text{!}$	a	b	$P(w_i = \text{!} \text{POS}) = ??$
$w_i = 0$	c	d	

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_i = \text{!} | \text{POS})$?

	POS	NEG
$w_i = \text{!}$	a	b
$w_i = 0$	c	d

$$P(w_i = \text{!} | \text{POS}) = a / (a + c)$$

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_i = \text{I} / \text{O} | \text{POS/NEG})$?

	POS	NEG
$w_i = \text{I}$	a	b
$w_i = \text{O}$	c	d

$$P(w_i = \text{I} | \text{POS}) = a / (a + c)$$

$$P(w_i = \text{O} | \text{POS}) = ??$$

$$P(w_i = \text{I} | \text{NEG}) = ??$$

$$P(w_i = \text{O} | \text{NEG}) = ??$$

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_i = \text{I} / \text{O} | \text{POS/NEG})$?

	POS	NEG
$w_i = \text{I}$	a	b
$w_i = \text{O}$	c	d

$$P(w_i = \text{I} | \text{POS}) = a / (a + c)$$

$$P(w_i = \text{O} | \text{POS}) = c / (a + c)$$

$$P(w_i = \text{I} | \text{NEG}) = b / (b + d)$$

$$P(w_i = \text{O} | \text{NEG}) = d / (b + d)$$

Naive Bayes Classification

example: positive/negative movie reviews

- Question: How do we estimate $P(w_2 = \text{I} / \text{O} | \text{POS/NEG})$?

	POS	NEG
$w_2 = \text{I}$	a	b
$w_2 = \text{O}$	c	d

$$P(w_2 = \text{I} | \text{POS}) = a / (a + c)$$

$$P(w_2 = \text{O} | \text{POS}) = c / (a + c)$$

$$P(w_2 = \text{I} | \text{NEG}) = b / (b + d)$$

$$P(w_2 = \text{O} | \text{NEG}) = d / (b + d)$$

- The value of a, b, c, and d would be different for different features $w_1, w_2, w_3, w_4, w_5, \dots, w_n$

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance **D**, predict positive (**POS**) if:

$$P(D|POS) \times P(POS) \geq P(D|NEG) \times P(NEG)$$

- Otherwise, predict negative (**NEG**)

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance \mathbf{D} , predict positive (**POS**) if:

$$P(POS) \times \prod_{i=1}^n P(w_i = D_i | POS) \geq P(NEG) \times \prod_{i=1}^n P(w_i = D_i | NEG)$$

- Otherwise, predict negative (**NEG**)

Naive Bayes Classification

example: positive/negative movie reviews

- Given instance $D = \mathbf{1011011}$, predict positive (**POS**) if:

$$P(w_1=\mathbf{1}|\text{POS}) \times P(w_2=\mathbf{0}|\text{POS}) \times P(w_3=\mathbf{1}|\text{POS}) \times P(w_4=\mathbf{1}|\text{POS}) \times P(w_5=\mathbf{0}|\text{POS}) \times P(w_6=\mathbf{1}|\text{POS}) \times P(w_7=\mathbf{1}|\text{POS}) \times P(\text{POS})$$

$$\geq$$

$$P(w_1=\mathbf{1}|\text{NEG}) \times P(w_2=\mathbf{0}|\text{NEG}) \times P(w_3=\mathbf{1}|\text{NEG}) \times P(w_4=\mathbf{1}|\text{NEG}) \times P(w_5=\mathbf{0}|\text{NEG}) \times P(w_6=\mathbf{1}|\text{NEG}) \times P(w_7=\mathbf{1}|\text{NEG}) \times P(\text{NEG})$$

- Otherwise, predict negative (**NEG**)

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- We still have a problem! What is it?

Naive Bayes Classification

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- Given instance $D = \mathbf{1011011}$, predict positive (**POS**) if:

$$P(w_1=\mathbf{1}|\text{POS}) \times P(w_2=\mathbf{0}|\text{POS}) \times P(w_3=\mathbf{1}|\text{POS}) \times P(w_4=\mathbf{1}|\text{POS}) \times P(w_5=\mathbf{0}|\text{POS}) \times \mathbf{P(w_6=\mathbf{1}|\text{POS})} \times P(w_7=\mathbf{1}|\text{POS}) \times P(\text{POS})$$

\geq

$$P(w_1=\mathbf{1}|\text{NEG}) \times P(w_2=\mathbf{0}|\text{NEG}) \times P(w_3=\mathbf{1}|\text{NEG}) \times P(w_4=\mathbf{1}|\text{NEG}) \times P(w_5=\mathbf{0}|\text{NEG}) \times P(w_6=\mathbf{1}|\text{NEG}) \times P(w_7=\mathbf{1}|\text{NEG}) \times P(\text{NEG})$$

- Otherwise, predict negative (**NEG**)

What if this never happens in the training data?

Smoothing Probability Estimates

- When estimating probabilities, we tend to ...
 - ▶ Over-estimate the probability of observed outcomes
 - ▶ Under-estimate the probability of unobserved outcomes
- The goal of smoothing is to ...
 - ▶ Decrease the probability of observed outcomes
 - ▶ Increase the probability of unobserved outcomes
- It's usually a good idea
- You probably already know this concept!

Smoothing Probability Estimates

- **YOU:** Are there mountain lions around here?
- **YOUR FRIEND:** Nope.
- **YOU:** How can you be so sure?
- **YOUR FRIEND:** Because I've been hiking here five times before and have never seen one.
- **YOU:** ????



Smoothing Probability Estimates

- **YOU:** Are there mountain lions around here?
- **YOUR FRIEND:** Nope.
- **YOU:** How can you be so sure?
- **YOUR FRIEND:** Because I've been hiking here five times before and have never seen one.
- **MOUNTAIN LION:** You should have learned about smoothing by taking INLS 613. Yum!



Add-One Smoothing

- Question: How do we estimate $P(w_2 = \text{I} / \text{O} | \text{POS/NEG})$?

	POS	NEG
$w_2 = \text{I}$	a	b
$w_2 = \text{O}$	c	d

$$P(w_2 = \text{I} | \text{POS}) = a / (a + c)$$

$$P(w_2 = \text{O} | \text{POS}) = c / (a + c)$$

$$P(w_2 = \text{I} | \text{NEG}) = b / (b + d)$$

$$P(w_2 = \text{O} | \text{NEG}) = d / (b + d)$$

Add-One Smoothing

- Question: How do we estimate $P(w_2 = \text{I} / \text{O} | \text{POS} / \text{NEG})$?

	POS	NEG
$w_2 = \text{I}$	$a + 1$	$b + 1$
$w_2 = \text{O}$	$c + 1$	$d + 1$

$$P(w_2 = \text{I} | \text{POS}) = ??$$

$$P(w_2 = \text{O} | \text{POS}) = ??$$

$$P(w_2 = \text{I} | \text{NEG}) = ??$$

$$P(w_2 = \text{O} | \text{NEG}) = ??$$

Add-One Smoothing

- Question: How do we estimate $P(w_2 = \text{I} / \text{O} | \text{POS} / \text{NEG})$?

	POS	NEG
$w_2 = \text{I}$	$a + 1$	$b + 1$
$w_2 = \text{O}$	$c + 1$	$d + 1$

$$P(w_2 = \text{I} | \text{POS}) = (a + 1) / (a + c + 2)$$

$$P(w_2 = \text{O} | \text{POS}) = (c + 1) / (a + c + 2)$$

$$P(w_2 = \text{I} | \text{NEG}) = (b + 1) / (b + d + 2)$$

$$P(w_2 = \text{O} | \text{NEG}) = (d + 1) / (b + d + 2)$$

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$$P(POS) \times \prod_{i=1}^n P(w_i = D_i | POS) \geq P(NEG) \times \prod_{i=1}^n P(w_i = D_i | NEG)$$

- Otherwise, predict negative (**NEG**)

Naive Bayes Classification

- Naive Bayes Classifiers are simple, effective, robust, and very popular
- Assumes that feature values are conditionally independent given the target class value
- This assumption does not hold in natural language
- Even so, NB classifiers are very powerful
- Smoothing is necessary in order to avoid zero probabilities