Minimum Perceptual Error Calculation for Perceptual Linearization

Bradley M. Hemminger

Department of Radiology,
University of North Carolina at Chapel Hill
Chapel Hill, NC 27599-7510
bmh@rad.unc.edu
(919) 966-2998

ABSTRACT

Perceptual linearization has been advocated for medical image presentation, both for the faithful reproduction of images, and for standardizing the appearance across different display devices. At this time, few studies have attempted to evaluate the effectiveness of a post-processing perceptual linearization step. We have recently analyzed the methods for computing perceptual linearization remappings and found them to introduce significant perceptual distortions in the linearization. We propose a quantitative method for analyzing the perceptual error in linearizations, called minimum perceptual error linearization (MPELIN).

1. INTRODUCTION

The perceptual linearization of video display monitors plays a significant role in medical image presentation[1,11,12]. First, it allows the maximum transfer of information to the human observer since each change in digital driving level of the display yields a perceptually equal step in perceived brightness by the human observer. Second, for an image to be perceived as similarly as possible when seen on different displays, the two displays must be standardized, which can be done when they have been perceptually linearized. Third, perceptual linearization allows us to calculate the perceived dynamic range of the display device, which allows comparing the maximum inherent contrast resolution of different devices.

Perceptual linearization was first suggested for medical image presentation by Pizer[1], and in follow-up work[2,3,4,5,6,7,8] at the University of North Carolina at Chapel Hill (UNC). To best visually present an image represented as digital data to the human observer, we would like to maximize the information transferred in mapping the digital driving levels to perceived brightness levels. Perceptually linearizing the mapping from the image data space to the human observer's visual sensory space most faithfully transmits changes in intensities in the image to the human observer.[3,6,9,10] This simply means that to the human observer, equal absolute changes in the input values to the display system should result in equal absolute changes in the perceived visual sensation.

The process of displaying an image on a video display monitor to the human observer is depicted in figure 1. This paradigm applies equally well to the display of images on film. Initially, an object, such as the human body, is scanned and the resulting signal (for instance tissue density) is represented on the computer as a matrix of points, called pixels. This scanning samples the original source data (continuous analog function) into discrete data (set of digital values). Each pixel is represented by a scalar value, usually in the range of 0 to 4096 for medical image data. These are the values referred to as *Recorded Intensities* in Figure 1. The second step is that some set of image processing operations, such as intensity windowing, or contrast enhancement may be performed on the *Recorded Intensities* resulting in the *Displayable Intensities*. These values are then scaled into *Digital Driving Levels* (DDLs), which must be in the range accepted by the *Digital to Analog Converter* (DAC) of the display system. This scaling is done by a table lookup operation, often referred to as a *Lookup Table* (LUT) or colormap table. LUTs are often used to do intensity windowing dynamically, or to implement a linearization LUT (these are sometimes called *gamma correction* curves). The output of the LUT goes to the DAC, which takes the input DDL and converts it to an analog voltage level which is used to drive the monitor at different luminance levels. The luminance generated by the monitor is then recorded and processed by the eye-brain human visual system, resulting in the sensation of brightness by the human observer.

From the standpoint of linearization there are two important relationships in this process, that of the DDLs of the computer's DAC versus the luminance of the monitor, and that of the monitor luminance versus the brightness perceived by the human observer. The first relationship of DAC to luminance will be referred to as *DACLUM*. The second relationship, that of luminance to perceived brightness, is best examined using a *luminance Contrast Sensitivity Function*, abbreviated as CSF in this paper. CSFs measure the change in luminance (ΔL) required for a target, so that it may be detected from the surround luminance (L) as a function of the surround luminance. More specifically, *Contrast Thresholds* (CT) are defined as $\Delta L/L$, while CSFs are defined as its reciprocal, i.e. $L/\Delta L$. CSFs in this paper will refer to $L/\Delta L$ versus L, while in vision literature, CSFs usually refer to $L/\Delta L$ versus spatial frequency of the target.

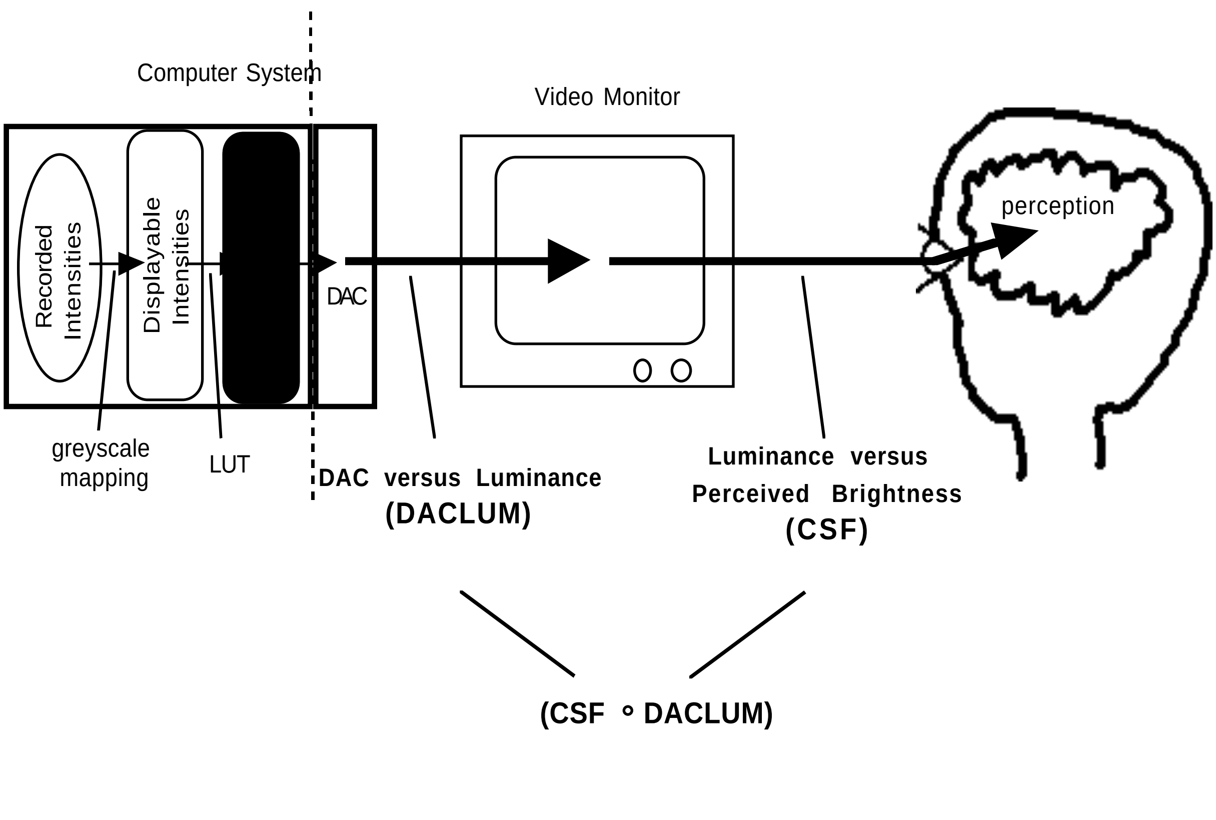

Figure 1. Diagram of components of Perceptual Linearization

If we think of the DACLUM and CSF curves as functions, and compose them on their common variable of luminance, we arrive at a CSF•DACLUM function that defines the overall effect of the

DACs, monitors, and human perception in the display system. The inverse of this function can be determined and used to remap the image values to perceptually linearize the relationship between the grey levels of the image in the computer and the sensation of brightness to the human observer.

Pizer, in his initial description of perceptual linearization gave both an intuitive and a formal analytical approach[1]. In the intuitive approach one calculates

$$L_i = L_{i-1} + (L_{i-1}) * (1 / (CSF(L_{i-1}))$$

until L_i reaches or exceeds the luminance of the maximum DDL. L_i represents the luminance at the ith DDL value, and (1 / (CSF(L_i)) the contrast threshold at luminance L_i. Thus in this formulation one simply steps 1 JND in luminance at each step, starting at the minimum luminance, until the maximum luminance is reached. The analytical formulation is given by Pizer[1] and the specifics of implementing the linearization by Cromartie[8]. Also an approximation that further simplifies the analytical solution is given by Ji[35]. In work on linearized color scales other authors have developed methods that supersample in the perceptual scale, and then choose the closest digital driving scale of the monitor[36,37].

Initial work at UNC used experimentally measure CSFs based on simple detection tasks on video monitors. Recently, several investigators have proposed using vision models as more general CSF predictors[9,15,16,24,25,26,35]. Specific parameters to the Barten visual model that match medical image presentations have been proposed as a standard[12,42]. This model is used in our linearization work and is referred to as the *reference Barten CSF*.

2. BACKGROUND

In all of the above linearization approaches, the final step takes a calculated desired luminance level and then selects the DDL that produces the luminance nearest in value to the desired luminance. Because there are limited discrete samples in the DDL range (256), and since they are often not distributed in a fashion matching the CSF function, errors may be introduced during this matching step. An example of this can be seen in Figure 2 which shows the monitor contrast threshold curve, the ideal contrast threshold curve from the reference Barten CSF, and a linearization function previously used in our laboratory. The linearization was based on the CIELUV visual model (i.e. 1/3 power law) for the CSF, and measurements of all the monitor luminance levels on our Sun Sparc2 for the DACLUM. It was calculated at 128 digital driving levels. As observed earlier, there is significant variation in the monitor step sizes at very low luminance levels, small spikes in the midrange, and larger variation (up to 200% changes in step sizes) in higher luminance levels. Surprisingly, though, the linearized curve is flawed as well. While it avoids much of the very low luminance levels entirely, for the remaining range up to $5x10^1$ cd/m^2 the variation in step sizes is larger than that of the default monitor (up to 300% change in step sizes). This is mainly due to the distribution of DAC luminance levels not matching the CSF well and the insufficient number of DDLs. Increasing the number of DAC output levels, and improving the DAC luminance levels distribution to better match the human visual response could improve this situation. Changes to the DAC distribution, however, require either change in manufacture of the DAC or an add-on circuit to provide a non-linear remapping of the DAC output voltages. Alternatively, a good method for computing the perceptual linearization remapping can be easily implemented in software by the end user.

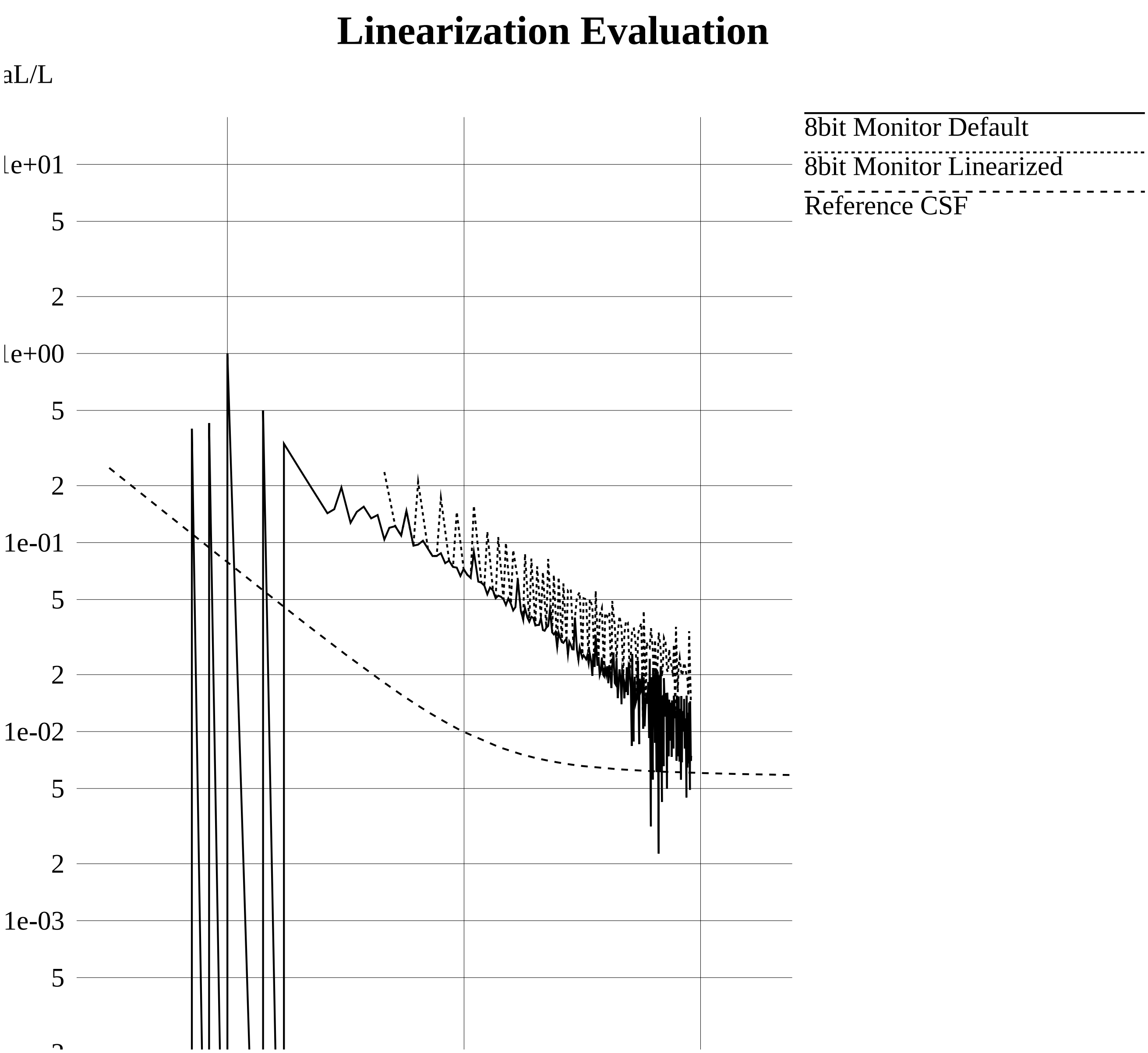

Figure 2. Contrast threshold curve for Sun Sparc Station monitor, resulting linearization for this monitor based on CIELUV algorithm, and human observer contrast threshold curve based on reference Barten visual model.

Another important issue is the number of DDLs in the resulting map. Linearization methods attempt to create mappings with each DDL step being an equal fraction of a JND step apart, often with the default implementation creating a table of steps being one JND apart. If the PDR range of the monitor is significantly less than the number of available DDLs on the DAC then we face the issue of whether to use more DDLs. Choosing not to do so means the contrast resolution must be downsampled to the smaller value of PDR rather than number of DDLs available. For instance, on a display system with a PDR of 80, we would have to downsample the input greyscale range of 4096 levels to just 80 levels. This coarse quantization of the input data may be undesirable. In order to use more DDLs one would have to resample the desired $(\mathrm{CSF} \bullet \mathrm{DACLUM})^{-1}$ curve, similar to the previously described supersampling methods.

None of the above techniques attempt to minimize the error introduced during this matching of $(\mathrm{CSF} \bullet \mathrm{DACLUM})^{-1}$ desired luminances and actual available discrete luminance levels. General solutions exist for the similar signal quantization problem of mapping a continuous variable into a discrete one[38,39]. This problems differs in that: (1) we have fixed non-uniform spacing of the luminances resulting from the DDLs; (2) we can use any or all of the DDLs in the mapping; (3) we want to minimize the equalness of the steps, not simply the distance from the result sample points to the desired ones; and (4) we would like to maximize the number of DDL levels steps used (to avoid over quantizing the input data) but not at the cost of compromising the accuracy of the linearization. We propose a solution to this problem, one that *minimizes the perceptual error* in the resulting linearization, and that describes the actual resulting PDR, or *Achievable PDR*.

III. METHODS

Perceptual linearization is based on the maintaining equality between digital driving level steps and perceived brightness steps. Equal changes in digital driving levels should confer equal changes in perceived brightness. The measure of change is usually defined as a contrast threshold, ΔL, where

$$\Delta L = \text{(change in luminance over an interval)/(mean luminance of interval)}$$

Thus, a measure of the perceptual linearity of a display system would be how equal the contrast thresholds resulting from adjacent DDLs are compared to the contrast threshold predicted by vision models. The contrast threshold of the monitor is easily calculated for the interval $[\mathrm{DDL}_i, \mathrm{DDD}_{i+1}]$ by

$$CT_{display} = (L_{i+1} - L_i)/((L_{i+1} + L_i)/2)$$

The contrast threshold for perceived brightness we base on human visual models, specifically the Barten model proposed by Blume[12] and Hemminger[42]. From these models we can calculate the predicted contrast threshold value, CT_{human}, at a specific luminance value. Computing contrast threshold values of the monitor at each interval, as described above, allows comparison with the visual model predicted contrast threshold at the mean luminance of each interval.

Perceptual linearization dictates maintaining equal step sizes across the range of the monitor. A given display system's contrast threshold as calculated from its step sizes may not be equal to those predicted by the human visual model. For instance, the step size of the display system may be four times the human observer threshold because there are too few DDLs. Or a very high resolution DAC may have step sizes that are subthreshold, that is, smaller than the human observer contrast threshold. In either case, the important consideration for perceptual linearization is that the ratio between display system contrast threshold and the human observer contrast threshold be the same across the range of the display system. Thus, we want to calculate

$$\text{ratio} = CT_{display} / CT_{human}$$

and measure how consistent it is across the range of the display system. Considering these ratio values to be members of a statistical population suggests using the statistical measure of variance to quantitatively define how similar the individual members are to their common mean. Since we have the complete population defined, the variance is simply the sum of the squares of the distance from each sample point to the mean of the population. Other measures might be considered, for instance, the standard deviation (the square root of the variance) which weights outliers less, and the mean population deviation (simple average of the distances to the mean). Because variance is commonly used to indicate the how similar the members are, and because strict adherence to the mean is better shown by the square of distance in the variance, we have chosen to use the variance of the population as our measure. This results in a single value that provides a quantitative measure of the perceptual error of a display system.

While variance provides a good handle on the error of the linearization, two other factors need to be defined to completely describe the linearization remapping. First, because the remapping may only use some of the digital driving levels available in order to produce a different transfer function curve on the display system, the resulting linearization often has fewer DDLs than the default system configuration which uses all DDLs. As a result, in general, the linearized remapping has larger contrast threshold steps because it must cover the same luminance range, but with fewer DDL steps. Finally, because there are adverse effects due to "contouring" when too few DDLs are used to cover a luminance range[34], it is important to quantify this effect by calculating the mean $CT_{display}/CT_{human}$ ratio. The number of DDL steps actually used is an important factor as the number of DDLs is often limited, both at the DAC level, and on some workstations by the window manager application.

Thus, a display can be characterized by three values, the variation of the ratio of the display contrast threshold step sizes versus the ideal human observer response, the mean of these ratios, and the total number of DDLs used in the linearization.

Because there may be many possible linearization remappings to consider in order to choose a good, or most optimal one according to some measure, it may be desirable to combine the mean and variance into one measure which can be used to compare two different linearizations. The variance represents how similar the shapes of the curves are, while the mean represents the shift in height between the curves. Ideally, we would like the $CT_{display}$ curve to be as similar as possible to the CT_{human} curve. We have adopted the following formula for relating the variance and the mean:

$$MPE(display,human) = K * variance(ratio) + mean(ratio)$$

where K is a constant of proportionality. The choice of K will depend on the image content in the presentation and the goal of the presentation. Based on the work showing contour artifacts when the $CT_{display}$ becomes larger than the CT_{human}, the mean ratio value is best when it is less than 1 (i.e. when the display contrast threshold steps are subthreshold human perception steps). If we knew that the CT_{human} values were exact for comparison with the $CT_{display}$ ones, then we might choose to weight subthreshold $CT_{display}$ values differently than suprathreshold ones, because they would be less likely to result in contour artifacts in the resulting display. However, while we know the shape of the CT_{human} curve is consistent across the range of parameters to the Barten visual model for medical image presentations, the exact position of the curve, and thus the absolute CT_{human} value, depends on these parameters, the image content and other viewing factors[42]. Thus, since we cannot differentiate without knowing the exact CT_{human} location, we have chosen not to weight the mean differently above and below the model predicted CT_{human} value.

4. RESULTS

We have applied MPELIN to several existing linearization remappings. In general remappings that visually appear better to the observer, generally have smaller variance values. We plan to conduct formal observer studies to evaluate the effect of variance with respect to observer performance and observer preference. An example output for the linearization depicted in figure 2, listing the variance, the mean, the number of DDLs, and the ratio at each level, is given in appendix 1.

5. DISCUSSION

To date, the linearization methods developed have been aimed at simply implementing a reasonable $(CSF \bullet DACLUM)^{-1}$ function. For many monitors the limitation of 8 bit DACs and sub-optimal luminance distributions (not matching the human observer CSF) mean that calculated linearizations may not be very optimal, and in some cases like figure 2, turn out to be worse than not linearizing. Developers of DACs need to better match their distributions to CSF distributions, both to improve the inherent perceptual linearity of their system, and to better allow for after-market perceptual linearization corrections of the display system. This paper presents a method for quantitatively calculating the minimum perceptual error of a linearization remapping of a display device based on the statistical variance of the ratio of contrast thresholds of the device versus the contrast thresholds of the human observer. It also provides a better description of the achievable dynamic range of a display device, based on the three quantitative measures: variance of the contrast threshold ratios, mean of the contrast threshold ratios, and number of DDL steps used.

6. ACKNOWLEDGMENTS

This work was supported in part by NIH grants P01-CA47982, R01-CA60193, and R01-CA44060.

7. REFERENCES

1. Pizer, SM, Chan, FH, "Evaluation of the number of discernible levels produced by a display", *Information Processing in Medical Imaging,* R DiPaola and E. Kahn, Eds., Editions INSERM, Paris, pp. 561-580, 1980.

2. Pizer SM, Intensity mappings: liearization, image-based, user-controlled", SPIE Display Technology II, Vol 271, pp21-27, 1981.

3. Pizer, SM, "Intensity mapping to linearize display devices", *Computer Graphics and Image Processing*, 17, pp 262-268, 1981.

4. Pizer SM, Johnston RE, Zimmerman JB, Chan FH, "Contrast perception with video displays", SPIE Vol 318, PACS for Medical Applications, pp223-230, 1982.

5. Pizer SM, "Psychovisual Issues in the Display of Medical Images", Proc NATO Advanced Studies Institute on Pictorial Information Systems in Medicine, 1985.

6. Johnston RE, Zimmerman JB, Rogers DC, Pizer SM, "Perceptual Standardization", Proceedings of Picture Archiving and Communcations Systems III", SPIE Vol 536, pp. 44-49, 1985.

7. Rogers DC, Johnston RE, Pizer SM, "Effect of Ambient Light o Electronically Displayed Medical Images As Measured by LUmikance Discrimination Thresholds", Journal Optical Society of America, 4, 5, May 1987.

8. Cromartie RC, Johnston RE, Pizer SM, Roger DC, "Standization of Electronic Display Devices Based on Human Perception", University of North Carolina at Chapel Hill, Technical Report 88-002, December 1987.

9. Ji, T, Roehrig, H, Blume, H, Guillen, J, "Optimizing the display function of display devices", *Proceedings Medical Imaging VI: Image Capture*, *Formatting*, and Display, Vol 1653, pp. 126-139, February 1992.

10. Briggs SJ, "Photometric technique for deriving a "best gamma" for displays", Advances in Display Technology, SPIE 199, pp 134-145, 1979.

11. Blume H, Roehrig H, Browne M, Ji TL, "Comparison of the Physical Performance of High Resolution CRT Displays and Films Recorded by Laser Image Printers and Display on Light-Boxes and the Need for a Display Standard", Proceedings of Medical Imaging: Image Capture, Formatting and Display, SPIE volume 1232, pp97-114, 1990.

12. Blume H, Daly S, Muka E, "Presentation of Medical Images on CRT displays A Renewed Proposal for a Display standard", Proceedings of Medical Imaging: Image Capture, Formatting and Display, SPIE volume 1897, pp215-231, 1993.

13. Naiman AV, Makous W, "Spatial non-linearities of grayscale CRT pixels", Proceedings of Human Vision, Visual Processing, and Digital DIsplay III, SPIE volume 1666, pp41-56, 1992.

14. Roehrig H, Blume H, Ji TL, Sundaresham MK, "Noise of the CRT Display System", Proceedings of Medical Imaging: Image Capture, Formatting and Display, SPIE volume 1897, pp232-245, 1993.

15. Barten PGJ, "Physical Model for the Contrast Sensitivity of the Human Eye", Proceedings of Human Vision, Visual Processing, and Digital Display III, SPIE vol 1666, 1992.

16. Daly S, "The visible differences predictor: an algorithm for the assessment of image fidelity", Proceedings of Human Vision, Visual Processing, and Digital Display III, SPIE vol 1666, 1992.

17. Murchison C (Editor), A Handbook of General Experimental Pschology, p769, Clark University Press, Worcester MA, 1934.

18. Hill CR, The Physics of Medical Imaging, p567-583, Webb S (Ed), IOP Publishing Ltd, Philadephia PA, 1988.

19. Wyszecki, G, Stiles, WS, Color Science: Concepts and Methods, Quantitative Data and Formulae, pp. 567-570, 2nd Edition, John Wiley and Sons, New York, 1982.

20. Cornsweet TN, Visual Perception, Academic Press, New York, 1970.

21. Haber RN, Hershenson M, The Psychology of Visual Perception, Holt Rinehart and Winston, New York NY, 1973.

22. Bijl P, Koenderink JJ, "Visibility of Elliptical Gaussian Blobs", Vision Research, Vol 33, No. 2, pp243-255, 1993.

23. Bijl P, Koenderink JJ, Toet A, "Visibility of Blobs with a Gaussian Luminance Profile", Vision Research, Vol 29, No. 4, pp447-456, 1989

24. Daly, S, "Quantitative Performance Assessment of an Algorithm for Determination of Image Fideltiy", S.I.D. Digetst of Technical Papers, V. 24, pp317-320, 1993.

25. Rogers JG, Carel WL, "Development of design criteria for sensor displays devices", Report HAC Ref. No. C6619, Huges Aircraft Company, Culver City, CA, Dec 1973.

26. Barten P, "Spatio-Temporal Model for the Contrast Sensitivity of the Human Eye and its Temporal Aspects", Proceedings of Human Vision, Visual Processing, and Digital Display IV, SPIE Vol 1913, pp2-14, 1993.

27. Lie I, "Visual Detection and Resolution as a Function of Adaption and Glare", Vision Research Vol 21, pp1793-1797, 1981.

28. Wildt GJ, Waarts RG, "The Influence of the Surround on Contrast Sensitivity", 2nd Intl. Conf. on Visual Psychophysics and Medical Imaging, Brussles, Belgium, July 2-3, 1981. (NY, USA:IEEE 1981).

29. Hood DC, Finkelstein MA, Sensitivity to light Handbook of Perception and Human Performance, Vol 1, eds Boff KR, Kaufman L, and Thomans JP, pp5-28, Wiley, 1986.

30. Kundel HL, "Visual Clues in the Interpretation of Medial Images", Journam of Clinical Neuropysiology, 7(4):472-483, 1990.

31. Burbeck CA, Pizer SM, "Object Representation by Cores", UNC-CS Technical Report 94-004, 1994.

32. DeGroot SG, Gebhard JW, "Pupil Size determined by adapting luminance", Journal of Optical Society America, 42, p492-295, 1952.

33. Kirk RE, Experimental Design: Procedures for Behavioral Sciences, pp261, Second Edition, Wadsworth, Belmont CA, 1982.

34. Sezan MI, Yip KL, Daly SJ, "Uniform Perceptual Quantization: Applications to Digital Radiolography", IEEE Transactions on Man, Machine, and Cybernetics, Vol SMC-17, No 4, pp622-634, 1987.

35. Ji T, Roehrig H, Blume H, Guilllen J, "Optimizing the Display Function of Display Devices", Proceedings of Medical Imaging VI: Image Capture, Formatting, and Display, eds Kim Y, SPIE vol 1653, 1992.

36. Robertson PK, "Color Image Display: Computational Framework Based on a Uniform Color Space", PhD thesis, Australian National University, April 1985, (CSIRO Tech Report No. 27)

37. Levkowitz, H, Herman GT, "Color Scales for Image Data", *IEEE Computer Graphics and Applications*, Vol 12, No 1, pp72-80, Jan 1992.

38. Jain AN, Fundamentals of Digital Image Processing, pp99-123, Prentice Hall, Englewood Cliffs NJ, 1989.

39. Max J, "Quantizing for Minimum Distortion", IRE Trans. on Info. Th., Vol IT-6, No. 1, pp7-12, March 1960.

40. Daly S, personal communication, Jan 1994.

41. Barrett HH, "Objective Assessment of Image Quality: Effect of Quantum Noise and Object Variability", J. Opt. Soc. Am., 7, pp1266-1278, 1990.

42. Hemminger BM, Johnston RE, Rolland JP, Muller K, "Perceptual Linearization of video display monitors for medical image presentation", Proceedings of Medical Imaging 1994: Image Capture, Formatting, and Display, eds Kim Y, SPIE vol 2264, 1994.

8. Appendix

Interval:	Luminances	L-Ave	L-Diff	%D-CSF	%H-CSF	%Ratio
[- 1]:	(0.212, 0.262)	0.237	0.050	21.10	2.05	9.30
[1- 2]:	(0.262, 0.294)	0.278	0.032	11.51	1.89	5.09
[2- 3]:	(0.294, 0.326)	0.310	0.032	10.32	1.78	4.80
[3- 4]:	(0.326, 0.374)	0.350	0.048	13.71	1.69	7.12
[4- 5]:	(0.374, 0.410)	0.392	0.036	9.18	1.59	4.77
[5- 6]:	(0.410, 0.496)	0.453	0.086	18.98	1.49	11.72
[6- 7]:	(0.496, 0.542)	0.519	0.046	8.86	1.40	5.34
[7- 8]:	(0.542, 0.588)	0.565	0.046	8.14	1.35	5.05
[8- 9]:	(0.588, 0.638)	0.613	0.050	8.16	1.29	5.30
[9- 10]:	(0.638, 0.748)	0.693	0.110	15.87	1.22	11.97
[10- 11]:	(0.748, 0.808)	0.778	0.060	7.71	1.16	5.63
[11- 12]:	(0.808, 0.868)	0.838	0.060	7.16	1.13	5.36
[12- 13]:	(0.868, 0.994)	0.931	0.126	13.53	1.07	11.60
[13- 14]:	(0.994, 1.066)	1.030	0.072	6.99	1.03	5.77
[14- 15]:	(1.066, 1.138)	1.102	0.072	6.53	1.01	5.44
[15- 16]:	(1.138, 1.212)	1.175	0.074	6.30	1.00	5.32
[16- 17]:	(1.212, 1.402)	1.307	0.190	14.54	0.96	14.10
[17- 18]:	(1.402, 1.488)	1.445	0.086	5.95	0.93	5.42
[18- 19]:	(1.488, 1.576)	1.532	0.088	5.74	0.91	5.35
[19- 20]:	(1.576, 1.756)	1.666	0.180	10.80	0.87	11.40
[20- 21]:	(1.756, 1.854)	1.805	0.098	5.43	0.84	5.50
[21- 22]:	(1.854, 1.948)	1.901	0.094	4.94	0.81	5.10
[22- 23]:	(1.948, 2.156)	2.052	0.208	10.14	0.78	11.99
[23- 24]:	(2.156, 2.264)	2.210	0.108	4.89	0.76	5.40
[24- 25]:	(2.264, 2.490)	2.377	0.226	9.51	0.75	11.75
[25- 26]:	(2.490, 2.608)	2.549	0.118	4.63	0.73	5.36
[26- 27]:	(2.608, 2.846)	2.727	0.238	8.73	0.71	11.31
[27- 28]:	(2.846, 3.028)	2.937	0.182	6.20	0.69	8.03
[28- 29]:	(3.028, 3.160)	3.094	0.132	4.27	0.67	5.33
[29- 30]:	(3.160, 3.284)	3.222	0.124	3.85	0.67	4.77
[30- 31]:	(3.284, 3.570)	3.427	0.286	8.35	0.65	11.76
[31- 32]:	(3.570, 3.706)	3.638	0.136	3.74	0.64	4.83
[32- 33]:	(3.706, 4.010)	3.858	0.304	7.88	0.63	11.54
[33- 34]:	(4.010, 4.156)	4.083	0.146	3.58	0.62	4.80
[34- 35]:	(4.156, 4.466)	4.311	0.310	7.19	0.61	10.83
[35- 36]:	(4.466, 4.644)	4.555	0.178	3.91	0.60	5.53
[36- 37]:	(4.644, 4.968)	4.806	0.324	6.74	0.59	10.46
[37- 38]:	(4.968, 5.144)	5.056	0.176	3.48	0.58	5.01
[38- 39]:	(5.144, 5.564)	5.354	0.420	7.84	0.57	12.74
[39- 40]:	(5.564, 5.750)	5.657	0.186	3.29	0.56	4.85
[40- 41]:	(5.750, 6.138)	5.944	0.388	6.53	0.55	10.77
[41- 42]:	(6.138, 6.310)	6.224	0.172	2.76	0.55	4.04
[42- 43]:	(6.310, 6.724)	6.517	0.414	6.35	0.54	10.72
[43- 44]:	(6.724, 6.924)	6.824	0.200	2.93	0.54	4.47
[44- 45]:	(6.924, 7.344)	7.134	0.420	5.89	0.53	10.11
[45- 46]:	(7.344, 7.572)	7.458	0.228	3.06	0.52	4.83
[46- 47]:	(7.572, 7.998)	7.785	0.426	5.47	0.52	9.54
[47- 48]:	(7.998, 8.452)	8.225	0.454	5.52	0.51	9.76
[48- 49]:	(8.452, 8.682)	8.567	0.230	2.68	0.51	4.28

[49- 50]:	(8.682, 9.030)	8.856	0.348	3.93	0.50	6.79	
[50- 51]:	(9.030, 9.496)	9.263	0.466	5.03	0.50	9.06	
[51- 52]:	(9.496, 10.014)	9.755	0.518	5.31	0.49	9.74	
[52- 53]:	(10.014, 10.260)	10.137		0.246	2.43	0.49	3.94
[53- 54]:	(10.260, 10.780)	10.520		0.520	4.94	0.49	9.11
[54- 55]:	(10.780, 11.320)	11.050		0.540	4.89	0.49	9.05
[55- 56]:	(11.320, 11.580)	11.450		0.260	2.27	0.48	3.69
[56- 57]:	(11.580, 12.160)	11.870		0.580	4.89	0.48	9.14
[57- 58]:	(12.160, 12.720)	12.440		0.560	4.50	0.48	8.40
[58- 59]:	(12.720, 13.000)	12.860		0.280	2.18	0.48	3.57
[59- 60]:	(13.000, 13.720)	13.360		0.720	5.39	0.47	10.38
[60- 61]:	(13.720, 14.060)	13.890		0.340	2.45	0.47	4.20
[61- 62]:	(14.060, 14.640)	14.350		0.580	4.04	0.47	7.63
[62- 63]:	(14.640, 15.300)	14.970		0.660	4.41	0.47	8.48
[63- 64]:	(15.300, 15.600)	15.450		0.300	1.94	0.46	3.20
[64- 65]:	(15.600, 16.260)	15.930		0.660	4.14	0.46	8.01
[65- 66]:	(16.260, 16.920)	16.590		0.660	3.98	0.46	7.72
[66- 67]:	(16.920, 17.640)	17.280		0.720	4.17	0.45	8.21
[67- 68]:	(17.640, 17.940)	17.790		0.300	1.69	0.45	2.75
[68- 69]:	(17.940, 18.820)	18.380		0.880	4.79	0.45	9.72
[69- 70]:	(18.820, 19.500)	19.160		0.680	3.55	0.44	7.02
[70- 71]:	(19.500, 19.960)	19.730		0.460	2.33	0.44	4.31
[71- 72]:	(19.960, 20.620)	20.290		0.660	3.25	0.44	6.44
[72- 73]:	(20.620, 21.460)	21.040		0.840	3.99	0.44	8.16
[73- 74]:	(21.460, 22.220)	21.840		0.760	3.48	0.43	7.02
[74- 75]:	(22.220, 22.660)	22.440		0.440	1.96	0.43	3.53
[75- 76]:	(22.660, 23.400)	23.030		0.740	3.21	0.43	6.45
[76- 77]:	(23.400, 24.300)	23.850		0.900	3.77	0.43	7.78
[77- 78]:	(24.300, 25.220)	24.760		0.920	3.72	0.43	7.69
[78- 79]:	(25.220, 25.740)	25.480		0.520	2.04	0.43	3.79
[79- 80]:	(25.740, 26.400)	26.070		0.660	2.53	0.42	4.96
[80- 81]:	(26.400, 27.320)	26.860		0.920	3.43	0.42	7.10
[81- 82]:	(27.320, 28.280)	27.800		0.960	3.45	0.42	7.20
[82- 83]:	(28.280, 29.100)	28.690		0.820	2.86	0.42	5.82
[83- 84]:	(29.100, 29.540)	29.320		0.440	1.50	0.42	2.59
[84- 85]:	(29.540, 30.500)	30.020		0.960	3.20	0.42	6.68
[85- 86]:	(30.500, 31.580)	31.040		1.080	3.48	0.42	7.38
[86- 87]:	(31.580, 32.760)	32.170		1.180	3.67	0.41	7.87
[87- 88]:	(32.760, 33.100)	32.930		0.340	1.03	0.41	1.50
[88- 89]:	(33.100, 34.540)	33.820		1.440	4.26	0.41	9.35
[89- 90]:	(34.540, 35.060)	34.800		0.520	1.49	0.41	2.64
[90- 91]:	(35.060, 36.100)	35.580		1.040	2.92	0.41	6.14
[91- 92]:	(36.100, 37.100)	36.600		1.000	2.73	0.41	5.69
[92- 93]:	(37.100, 38.120)	37.610		1.020	2.71	0.41	5.67
[93- 94]:	(38.120, 39.460)	38.790		1.340	3.45	0.41	7.52
[94- 95]:	(39.460, 40.540)	40.000		1.080	2.70	0.40	5.68
[95- 96]:	(40.540, 41.240)	40.890		0.700	1.71	0.40	3.25
[96- 97]:	(41.240, 42.500)	41.870		1.260	3.01	0.40	6.48
[97- 98]:	(42.500, 43.320)	42.910		0.820	1.91	0.40	3.76
[98- 99]:	(43.320, 44.320)	43.820		1.000	2.28	0.40	4.69
[99- 100]:	(44.320, 45.800)	45.060		1.480	3.28	0.40	7.22
[100- 101]:	(45.800, 47.100)	46.450		1.300	2.80	0.40	6.02
[101- 102]:	(47.100, 48.020)	47.560		0.920	1.93	0.40	3.86

[102- 103]:	(48.020, 49.080)	48.550	1.060	2.18	0.40	4.50
[103- 104]:	(49.080, 50.620)	49.850	1.540	3.09	0.40	6.81
[104- 105]:	(50.620, 52.120)	51.370	1.500	2.92	0.39	6.40
[105- 106]:	(52.120, 53.220)	52.670	1.100	2.09	0.39	4.30
[106- 107]:	(53.220, 54.320)	53.770	1.100	2.05	0.39	4.20
[107- 108]:	(54.320, 55.800)	55.060	1.480	2.69	0.39	5.85
[108- 109]:	(55.800, 57.100)	56.450	1.300	2.30	0.39	4.88
[109- 110]:	(57.100, 58.360)	57.730	1.260	2.18	0.39	4.58
[110- 111]:	(58.360, 59.500)	58.930	1.140	1.93	0.39	3.96
[111- 112]:	(59.500, 61.220)	60.360	1.720	2.85	0.39	6.32
[112- 113]:	(61.220, 61.940)	61.580	0.720	1.17	0.39	2.01
[113- 114]:	(61.940, 64.160)	63.050	2.220	3.52	0.39	8.08
[114- 115]:	(64.160, 65.200)	64.680	1.040	1.61	0.39	3.15
[115- 116]:	(65.200, 66.660)	65.930	1.460	2.21	0.39	4.73
[116- 117]:	(66.660, 68.300)	67.480	1.640	2.43	0.39	5.30
[117- 118]:	(68.300, 69.740)	69.020	1.440	2.09	0.38	4.42
[118- 119]:	(69.740, 71.120)	70.430	1.380	1.96	0.38	4.10
[119- 120]:	(71.120, 72.600)	71.860	1.480	2.06	0.38	4.37
[120- 121]:	(72.600, 74.280)	73.440	1.680	2.29	0.38	4.97
[121- 122]:	(74.280, 75.900)	75.090	1.620	2.16	0.38	4.64
[122- 123]:	(75.900, 77.420)	76.660	1.520	1.98	0.38	4.19
[123- 124]:	(77.420, 78.900)	78.160	1.480	1.89	0.38	3.97
[124- 125]:	(78.900, 80.120)	79.510	1.220	1.53	0.38	3.03
[125- 126]:	(80.120, 82.840)	81.480	2.720	3.34	0.38	7.78
[126- 127]:	(82.840, 84.040)	83.440	1.200	1.44	0.38	2.79

DDLs=128
Zero-based statistics
Mean=6.493099 Var=7.068256 Stddev=2.658619 DistVar=2.196183
Original based Mean=7.493099