# Language Models for Information Retrieval

Jaime Arguello
INLS 509: Information Retrieval
jarguell@email.unc.edu

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# Outline

Introduction to language modeling

Language modeling for information retrieval

Query-likelihood Retrieval Model

Smoothing

Pseudo-relevance feedback and priors

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Introduction to language modeling

Language modeling for information retrieval

Query-likelihood Retrieval Model

Smoothing

Pseudo-relevance feedback and priors

# What is a language model?

"The goal of a language model is to assign a probability to a sequence of words by means of a probability distribution"

--Wikipedia

# Motivating Example

## speech-to-text conversion for mobile search

"jaime arguello information retrieval at the university of north carolina at chapel hill"

DID YOU MEAN:

"hi man whale information retrieval at the university of !? north carolina at chapel hill"?

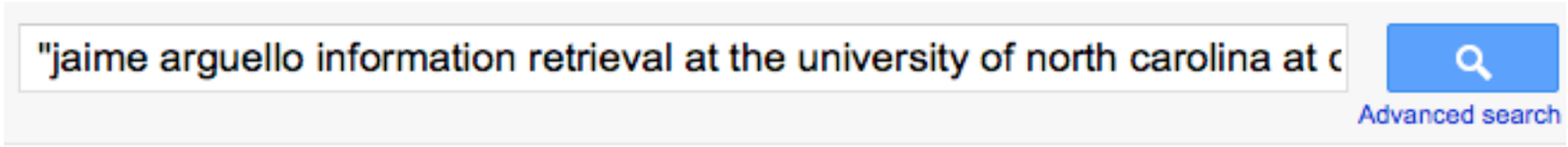
# Motivating Example
## speech-to-text conversion for mobile search

- Which sequence of words is more likely to occur?
  - jaime arguello information retrieval at the university of north carolina at chapel hill
  - hi man whale information retrieval at the university of north carolina at chapel hill

# Motivating Example

## speech-to-text conversion for mobile search

- Let's see what the internets say!




⚠ No results found for **"jaime arguello information retrieval at the university of north carolina at chapel hill"**.

Results for **jaime arguello information retrieval at the university of north carolina at chapel hill** (without quotes):

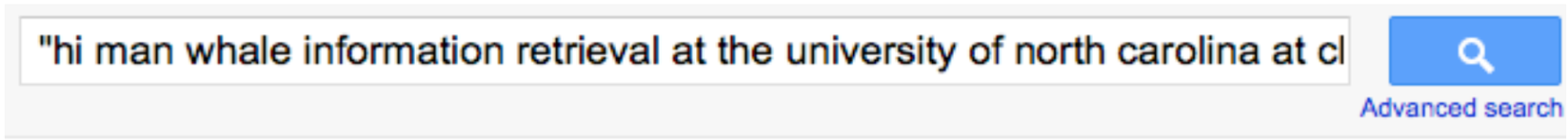
Three scholars join SILS faculty | sils.**unc**.edu
sils.unc.edu/news/2011/new-faculty
May 31, 2011 – The School of **Information** and Library Science (SILS) at the **University of North Carolina at Chapel Hill** welcomes scholars **Jaime Arguello**, ...

**Jaime Arguello**
www.ils.unc.edu/~jarguello/
**Jaime Arguello** does research on **information retrieval** at the School of ...

# Motivating Example

## speech-to-text conversion for mobile search

- Let's see what the internets say!




⚠ No results found for **"hi man whale information retrieval at the university of north carolina at chapel hill"**.

Results for **hi man whale information retrieval at the university of north carolina at chapel hill** (without quotes):

The Coming Of Age Of American Business: Three Centuries Of ...
www.questia.com/googleScholar.qst?docId=12003164
by EP Douglass - Cited by 13 - Related articles
and recording, or by any **information** storage and **retrieval** system, without permission in .... department of The **University of North Carolina at Chapel Hill** to sub- ...

[PDF] Effective Methods for Studying **Information** Seeking and Use
www.asis.org/SIG/SIGUSE/SIGUSE-Proceedings.2001.pdf
File Format: PDF/Adobe Acrobat - Quick View
Claudia Gollop, **University of North Carolina at Chapel Hill**. Jane Greenberg ...
sources present in one's context/situation, **retrieving information** from available ...

# Motivating Example
## speech-to-text conversion for mobile search

- This example raises some questions:
  - How could the system predict an output that has never occurred before (according to the web)?
  - How could the system assign this output a non-zero probability?
  - And, why would it predict “hi man whale” over “jaime arguello”?
- Answer: statistical language modeling!

# What is a language model?

- To understand what a language model is, we have to understand what a probability distribution is

- To understand what a probability distribution is, we have to understand what a discrete random variable is

# What is a discrete random variable?

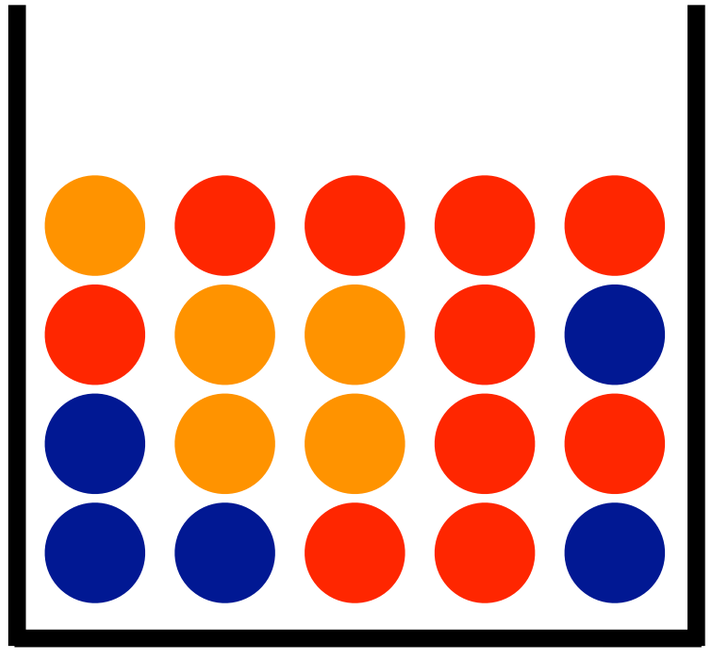
- Let A denote a discrete random variable
- A is a discrete random variable if:
  - A describes an event with a finite number of possible outcomes (this property makes the random variable discrete)
  - A describes an event whose outcome has some degree of uncertainty (this property makes the variable random)

# Discrete Random Variables
## examples

- A = it will rain tomorrow
- A = the coin-flip will show heads
- A = you will win the lottery in your lifetime
- A = the 2023 US president will be female
- A = you have the flu
- A = you will find the next couple of slides fascinating

# What is a probability distribution?

- A probability distribution gives the probability of each possible outcome of a random variable



- P(RED) = probability that you will reach into this bag and pull out a red ball

- P(BLUE) = probability that you will reach into this bag and pull out a blue ball

- P(ORANGE) = probability that you will reach into this bag and pull out an orange ball

# What is a probability distribution?

- For it to be a probability distribution, two conditions must be satisfied:
  - the probability assigned to each possible outcome must be between 0 and 1 (inclusive)
  - the sum of probabilities across outcomes must be 1

$$0 \leq P(\text{RED}) \leq 1$$

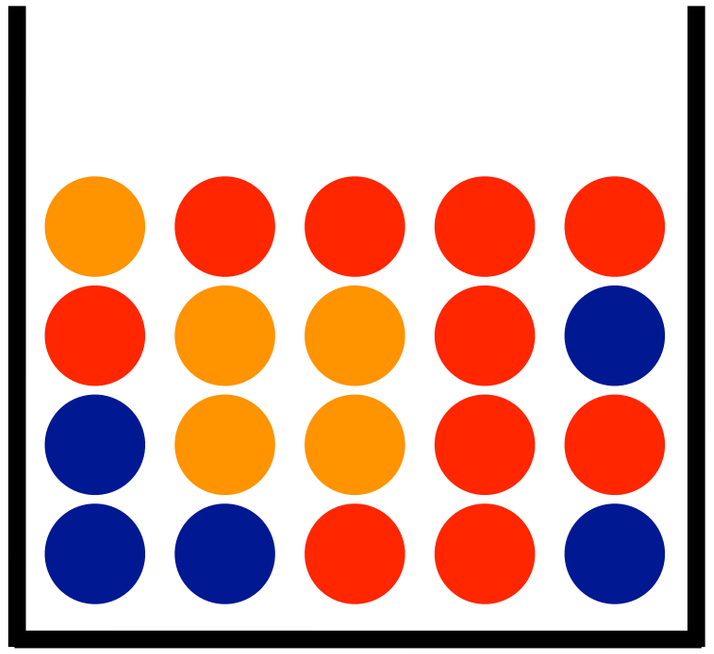
$$0 \leq P(\text{BLUE}) \leq 1$$

$$0 \leq P(\text{ORANGE}) \leq 1$$

$$P(\text{RED}) + P(\text{BLUE}) + P(\text{ORANGE}) = 1$$

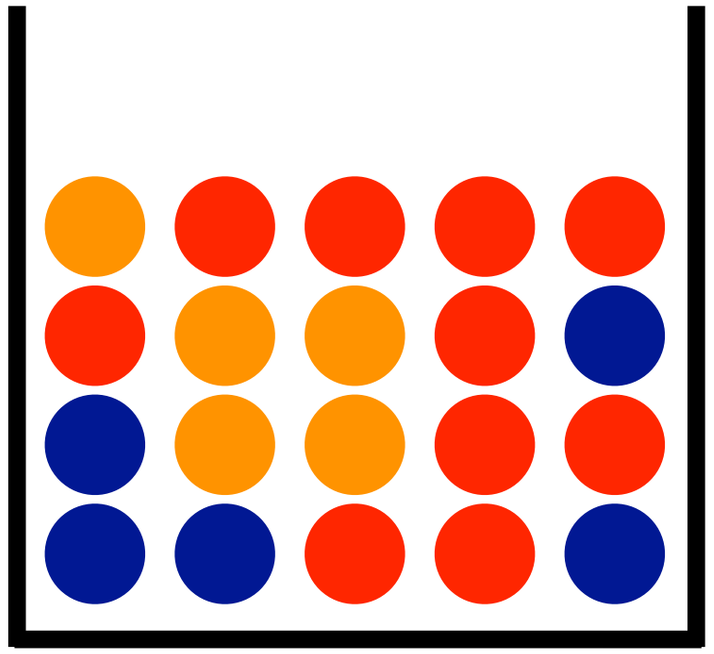
# Estimating a Probability Distribution

- Let's estimate these probabilities based on what we know about the contents of the bag
- P(RED) = ?
- P(BLUE) = ?
- P(ORANGE) = ?



# Estimating a Probability Distribution

- Let's estimate these probabilities based on what we know about the contents of the bag
- P(RED) = 10/20 = 0.5
- P(BLUE) = 5/20 = 0.25
- P(ORANGE) = 5/20 = 0.25
- P(RED) + P(BLUE) + P(ORANGE) = 1.0



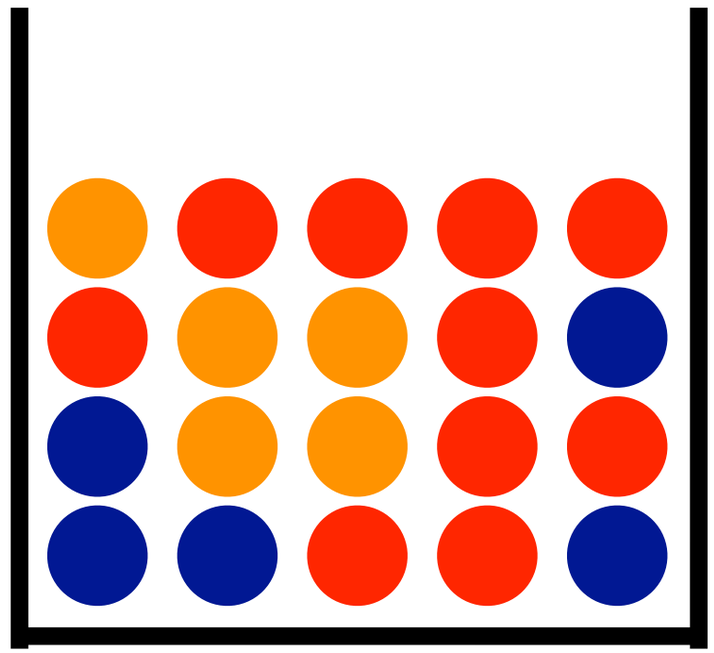
# What can we do with a probability distribution?

- We can assign probabilities to different outcomes
- I reach into the bag and pull out an orange ball. What is the probability of that happening?
- I reach into the bag and pull out two balls: one red, one blue. What is the probability of that happening?
- What about three orange balls?

P(RED) = 0.5

P(BLUE) = 0.25

P(ORANGE) = 0.25

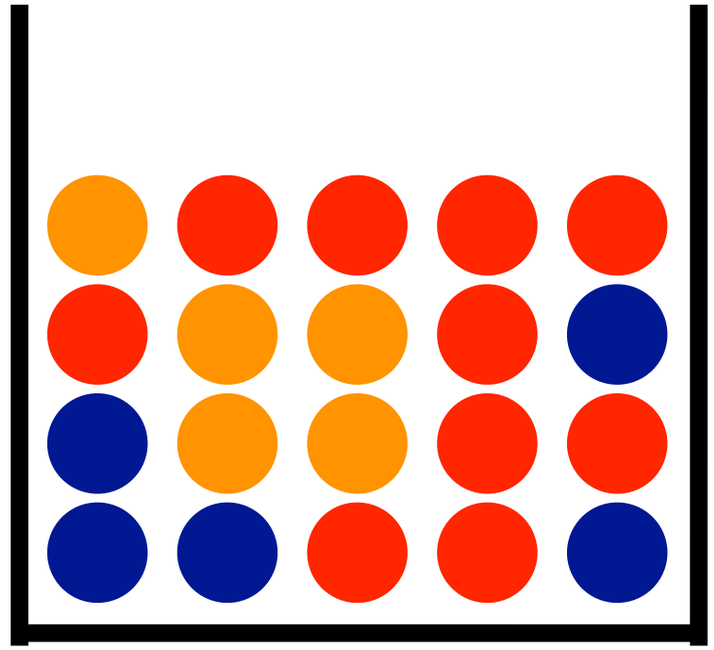


# What can we do with a probability distribution?

- If we assume that each outcome is independent of previous outcomes, then the probability of a sequence of outcomes is calculated by multiplying the individual probabilities

- Note: we're assuming that when you take out a ball, you put it back in the bag before taking another one out

P(RED) = 0.5
P(BLUE) = 0.25
P(ORANGE) = 0.25



# What can we do with a probability distribution?

- P( ● ) = 0.25
- P( ● ) = 0.5
- P( ● ● ● ) = 0.25 x 0.25 x 0.25
- P( ● ● ● ) = 0.25 x 0.25 x 0.25
- P( ● ● ● ) = 0.25 x 0.50 x 0.25
- P( ● ● ● ● ) = 0.25 x 0.50 x 0.25 x 0.50

P(RED) = 0.5
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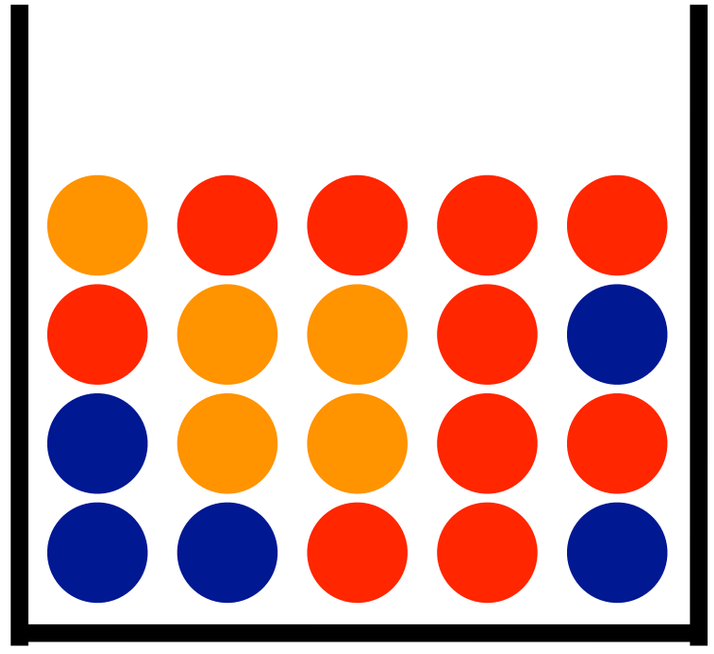


# Now, let's return to our example

"jaime arguello information retrieval at the university of north carolina at chapel hill"

DID YOU MEAN:

"hi man whale information retrieval at the university of north carolina at chapel hill"?

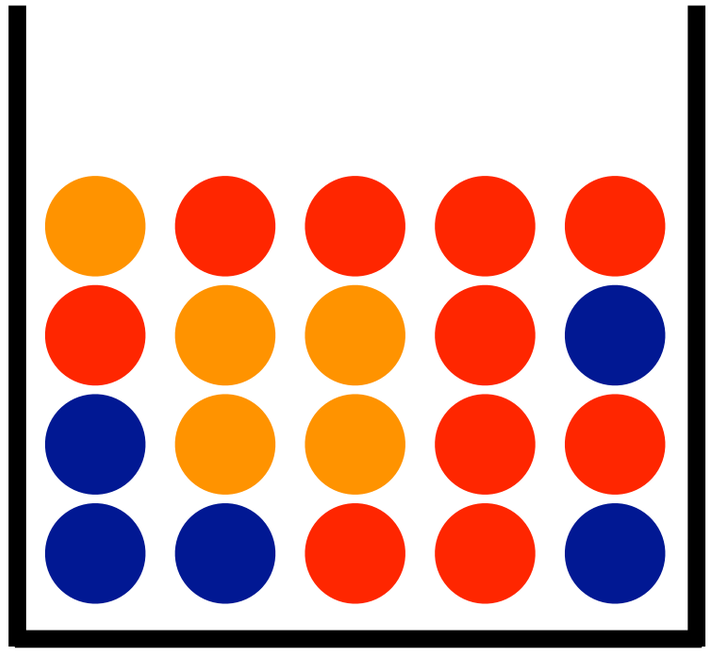
!?

# Now, let's return to our example

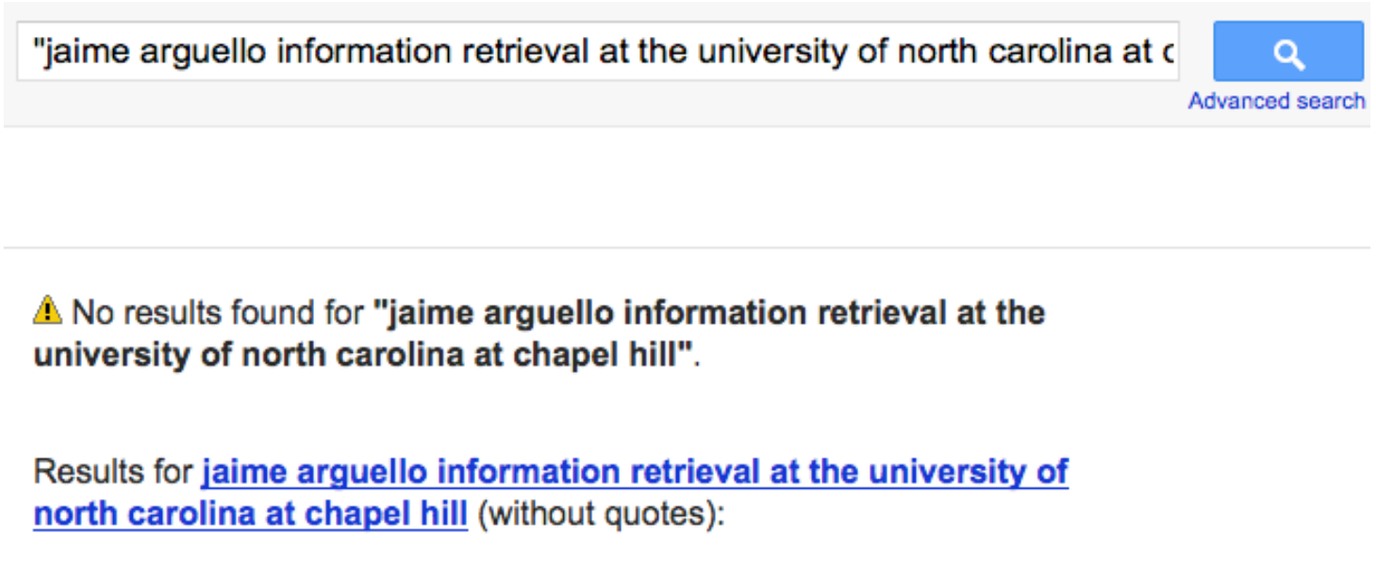
"jaime arguello information retrieval at the university of north carolina at chapel hill"

DID YOU MEAN:
"hi man whale information retrieval at the university of north carolina at chapel hill"?

- We want to assign a probability to a particular sequence of words
- We want to use a "bag of something" to do it (similar to our bag of colored balls)
- What should go in the bag? Sequences of words? Individual words?



# Why not (long) sequences of words?




- Believe it or not, there is not enough data to estimate these probabilities accurately!
- Many sequences of words have never been observed!
- Solution: unigram language model

# Unigram Language Model

- Defines a probability distribution over individual words
  - P(university) = 2/20
  - P(of) = 4/20
  - P(north) = 2/20
  - P(carolina) = 1/20
  - P(at) = 4/20
  - P(chapel) = 3/20
  - P(hill) = 4/20

university university
of of of of
north north
carolina
at at at at
chapel chapel chapel
hill hill hill hill

# Unigram Language Model

- It is called a unigram language model because we estimate (and predict) the likelihood of each word independent of any other word
- Assumes that words are independent!
  - The probability of seeing “tarheels” is the same, even if the preceding word is “carolina”
- Other language models take context into account
- Those work better for applications like speech recognition or automatic language translation
- Unigram models work well for information retrieval

# Unigram Language Model

- Sequences of words can be assigned a probability by multiplying their individual probabilities:

P(university of north carolina) =

P(university) x P(of) x P(north) x P(carolina) =

(2/20) x (4/20) x (2/20) x (1/20) = 0.0001

P(chapel hill) =

P(chapel) x P(hill) =

(3/20) x (4/20) = 0.03

# Unigram Language Model

- There are two important steps in language modeling
  - estimation: observing text and estimating the probability of each word
  - prediction: using the language model to assign a probability to a span of text

# Unigram Language Model

- Any span of text can be used to estimate a language model
- And, given a language model, we can assign a probability to any span of text
  - a word
  - a sentence
  - a document
  - a corpus
  - the entire web

# Unigram Language Model Estimation

- General estimation approach:
  - tokenize/split the text into terms
  - count the total number of term occurrences (***N***)
  - count the number of occurrences of each term ($tf_t$)
  - assign term ***t*** a probability equal to

$$P_t = \frac{tf_t}{N}$$

# IMDB Corpus

## language model estimation (top 20 terms)

| term | tf | N | P(term) | term | tf | N | P(term) |
|---|---|---|---|---|---|---|---|
| the | 1586358 | 36989629 | 0.0429 | year | 250151 | 36989629 | 0.0068 |
| a | 854437 | 36989629 | 0.0231 | he | 242508 | 36989629 | 0.0066 |
| and | 822091 | 36989629 | 0.0222 | movie | 241551 | 36989629 | 0.0065 |
| to | 804137 | 36989629 | 0.0217 | her | 240448 | 36989629 | 0.0065 |
| of | 657059 | 36989629 | 0.0178 | artist | 236286 | 36989629 | 0.0064 |
| in | 472059 | 36989629 | 0.0128 | character | 234754 | 36989629 | 0.0063 |
| is | 395968 | 36989629 | 0.0107 | cast | 234202 | 36989629 | 0.0063 |
| i | 390282 | 36989629 | 0.0106 | plot | 234189 | 36989629 | 0.0063 |
| his | 328877 | 36989629 | 0.0089 | for | 207319 | 36989629 | 0.0056 |
| with | 253153 | 36989629 | 0.0068 | that | 197723 | 36989629 | 0.0053 |

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| with | 253153 | 36989629 | 0.0068 | that | 197723 | 36989629 | 0.0053 |

- What is the probability associated with "artist of the year"?

# IMDB Corpus

## language model estimation (top 20 terms)

| term | tf | N | P(term) | term | tf | N | P(term) |
|---|---|---|---|---|---|---|---|
| the | 1586358 | 36989629 | 0.0429 | year | 250151 | 36989629 | 0.0068 |
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| with | 253153 | 36989629 | 0.0068 | that | 197723 | 36989629 | 0.0053 |

- What is more probable: "artist of the year" or "movie to the year?"

# IMDB Corpus

## language model estimation (top 20 terms)

| term | tf | N | P(term) | term | tf | N | P(term) |
|---|---|---|---|---|---|---|---|
| the | 1586358 | 36989629 | 0.0429 | year | 250151 | 36989629 | 0.0068 |
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| with | 253153 | 36989629 | 0.0068 | that | 197723 | 36989629 | 0.0053 |

- What is the most probable sequence "artist of the ____"?

# Motivating Example
## speech-to-text conversion for mobile search

- This example raises some questions:
  - How could the system predict an output that has never occurred before (according to the web)?
  - In other words, how could the system assign this output a non-zero probability?
  - And, why would it predict “hi man whale” over “jaime arguello”?
- Answer: language modeling!

# Outline

Introduction to language modeling

Language modeling for information retrieval

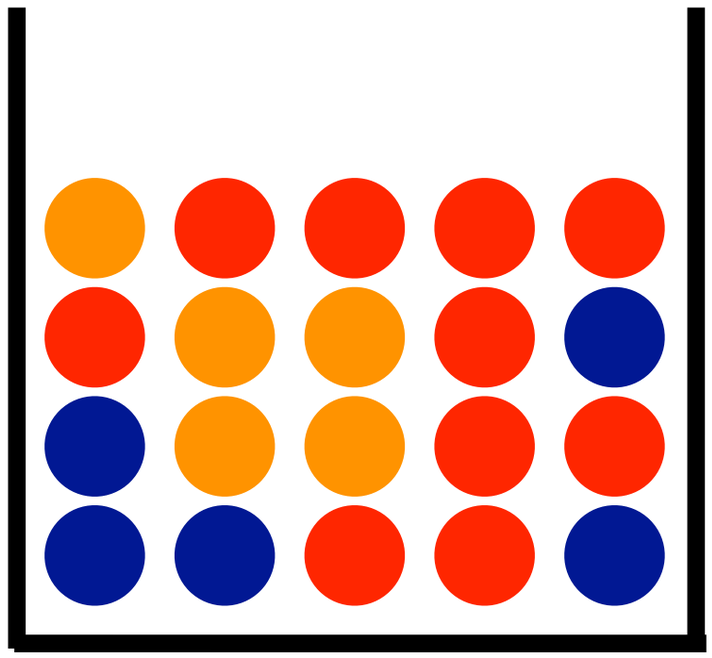
Query-likelihood Retrieval Model

Smoothing

Pseudo-relevance feedback and priors

# Language Models

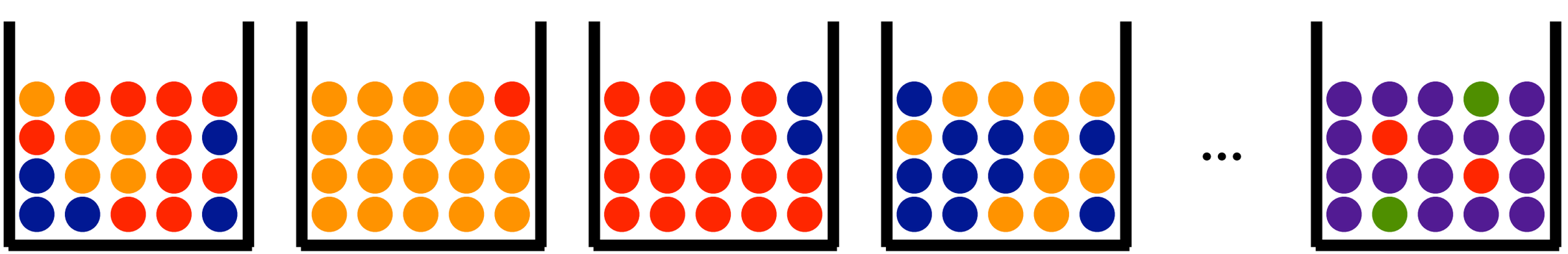
- A language model is a probability distribution defined over a particular vocabulary
- In this analogy, each color represents a vocabulary term and each ball represents a term occurrence in the text used to estimate the language model



P(RED) = 0.5
P(BLUE) = 0.25
P(ORANGE) = 0.25

# Topic Models

- We can think of a topic as being defined by a language model
- A high-probability of seeing certain words and a low-probability of seeing others



movies

P(RED) = 0.5
P(BLUE) = 0.25
P(ORANGE) = 0.25

politics

P(RED) = 0.05
P(BLUE) = 0.00
P(ORANGE) = 0.95

sports

P(RED) = 0.90
P(BLUE) = 0.10
P(ORANGE) = 0.00

music

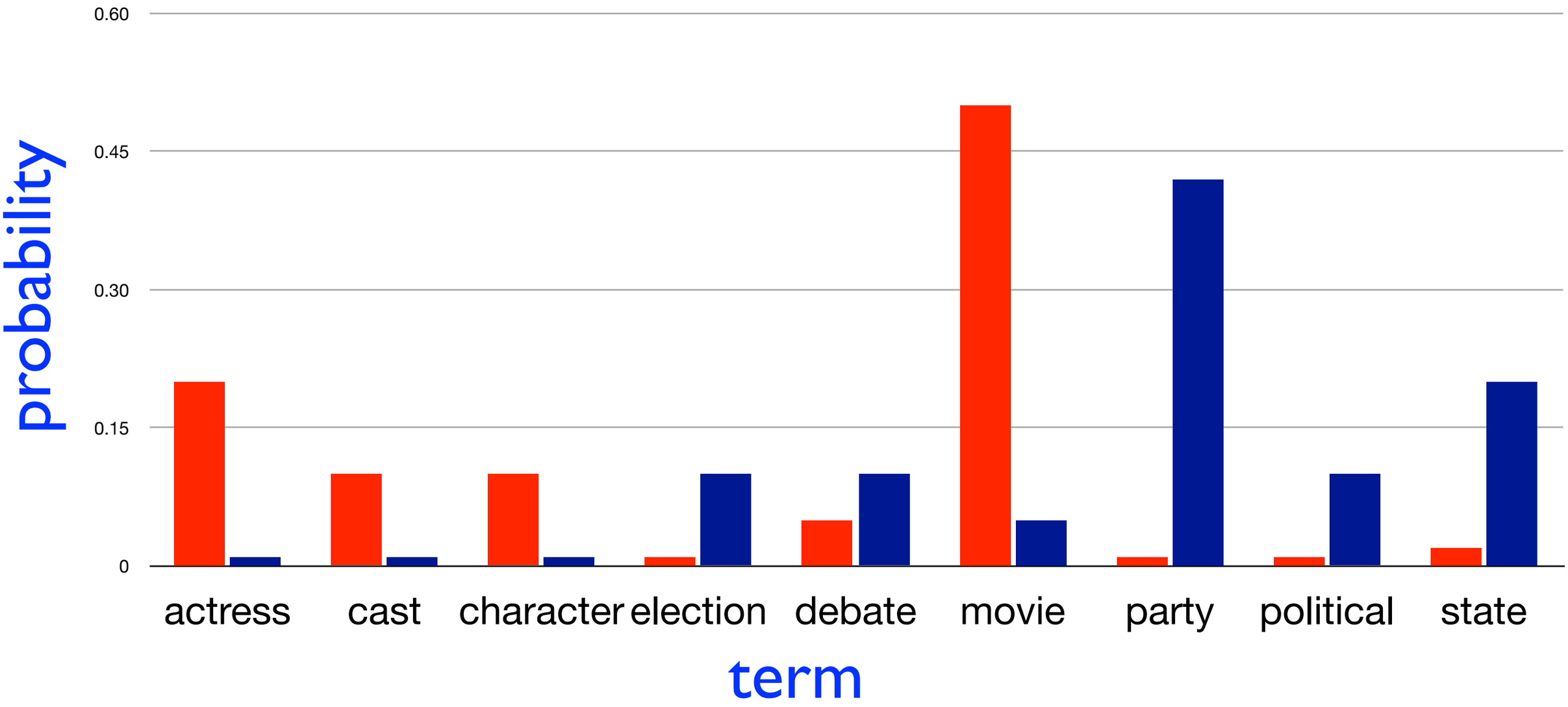
P(RED) = 0.00
P(BLUE) = 0.50
P(ORANGE) = 0.50

nature

P(RED) = 0.10
P(BLUE) = 0.80
P(ORANGE) = 0.10

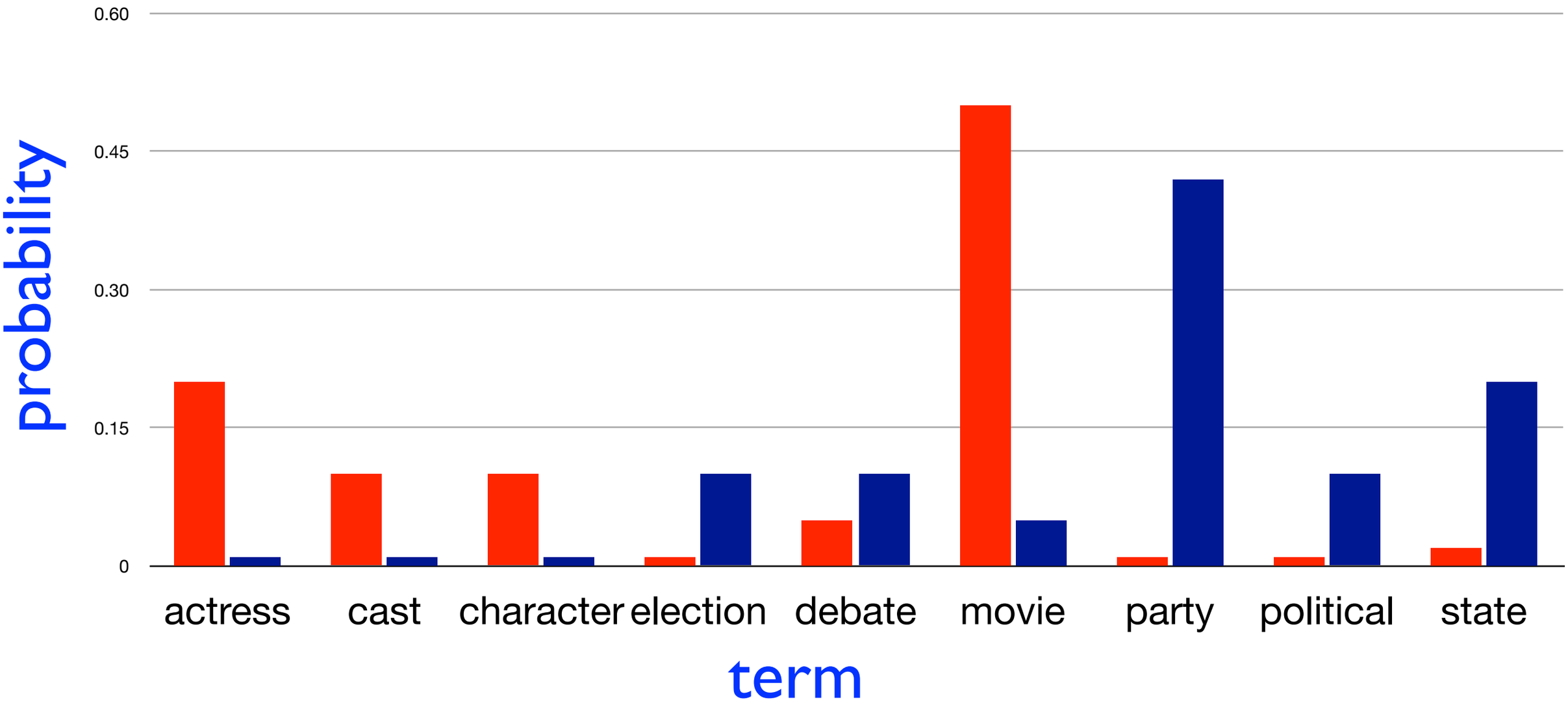
# Topic Models

## ??? vs. ???



# Topic Models

## movies vs. politics

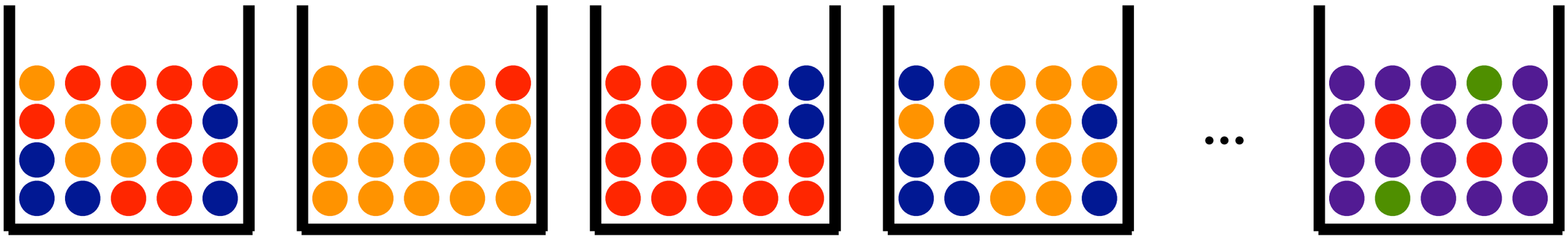


# Topical Relevance

- Many factors affect whether a document satisfies a particular user's information need
- Topicality, novelty, freshness, authority, formatting, reading level, assumed level of expertise, etc.
- Topical relevance: the document is on the same topic as the query
- User relevance: everything else!
- Remember, our goal right now is to predict topical relevance

# Document Language Models

- The topic (or topics) discussed in a particular document can be captured by its language model



movies | politics | sports | music | ... | nature

P(RED) = 0.5
P(BLUE) = 0.25
P(ORANGE) = 0.25

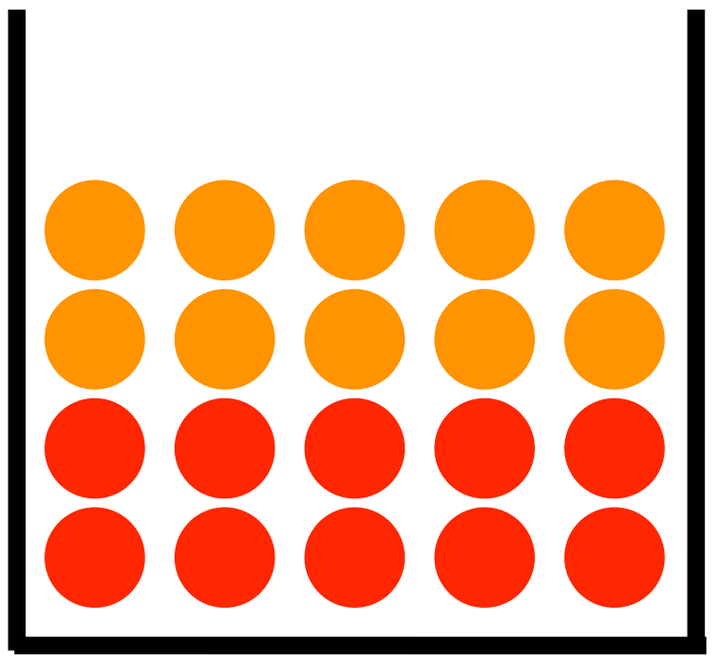
P(RED) = 0.05
P(BLUE) = 0.00
P(ORANGE) = 0.95

P(RED) = 0.90
P(BLUE) = 0.10
P(ORANGE) = 0.00

P(RED) = 0.00
P(BLUE) = 0.50
P(ORANGE) = 0.50

P(RED) = 0.10
P(BLUE) = 0.80
P(ORANGE) = 0.10

Document $D_{232}$



What is this document about?

# Document Language Models

- Estimating a document's language model:

  1. tokenize/split the document text into terms

  2. count the number of times each term occurs ($tf_{t,D}$)

  3. count the total number of term occurrences ($N_D$)

  4. assign term ***t*** a probability equal to:

$$\frac{tf_{t,D}}{N_D}$$

# Document Language Models

- The language model estimated from document ***D*** is sometimes denoted as:

$$\theta_D$$

- The probability given to term ***t*** by the language model estimated from document ***D*** is sometimes denoted as:

$$P(t|D) = P(t|\theta_D) = \frac{tf_{t,D}}{N_D}$$

# Document Language Models

- Movie: Rocky (1976)
- Plot:

Rocky Balboa is a struggling boxer trying to make the big time. Working in a meat factory in Philadelphia for a pittance, he also earns extra cash as a debt collector. When heavyweight champion Apollo Creed visits Philadelphia, his managers want to set up an exhibition match between Creed and a struggling boxer, touting the fight as a chance for a "nobody" to become a "somebody". The match is supposed to be easily won by Creed, but someone forgot to tell Rocky, who sees this as his only shot at the big time. Rocky Balboa is a small-time boxer who lives in an apartment in Philadelphia, Pennsylvania, and his career has so far not gotten off the canvas. Rocky earns a living by collecting debts for a loan shark named Gazzo, but Gazzo doesn't think Rocky has the viciousness it takes to beat up deadbeats. Rocky still boxes every once in a while to keep his boxing skills sharp, and his ex-trainer, Mickey, believes he could've made it to the top if he was willing to work for it. Rocky, goes to a pet store that sells pet supplies, and this is where he meets a young woman named Adrian, who is extremely shy, with no ability to talk to men. Rocky befriends her. Adrain later surprised Rocky with a dog from the pet shop that Rocky had befriended. Adrian's brother Paulie, who works for a meat packing company, is thrilled that someone has become interested in Adrian, and Adrian spends Thanksgiving with Rocky. Later, they go to Rocky's apartment, where Adrian explains that she has never been in a man's apartment before. Rocky sets her mind at ease, and they become lovers. Current world heavyweight boxing champion Apollo Creed comes up with the idea of giving an unknown a shot at the title. Apollo checks out the Philadelphia boxing scene, and chooses Rocky. Fight promoter Jergens gets things in gear, and Rocky starts training with Mickey. After a lot of training, Rocky is ready for the match, and he wants to prove that he can go the distance with Apollo. The 'Italian Stallion', Rocky Balboa, is an aspiring boxer in downtown Philadelphia. His one chance to make a better life for himself is through his boxing and Adrian, a girl who works in the local pet store. Through a publicity stunt, Rocky is set up to fight Apollo Creed, the current heavyweight champion who is already set to win. But Rocky really needs to triumph, against all the odds...

# Document Language Models

## language model estimation (top 20 terms)

| *term* | $tf_{t,D}$ | $N_D$ | *P(term\|D)* | *term* | $tf_{t,D}$ | $N_D$ | *P(term\|D)* |
|---|---|---|---|---|---|---|---|
| a | 22 | 420 | 0.05238 | creed | 5 | 420 | 0.01190 |
| rocky | 19 | 420 | 0.04524 | philadelphia | 5 | 420 | 0.01190 |
| to | 18 | 420 | 0.04286 | has | 4 | 420 | 0.00952 |
| the | 17 | 420 | 0.04048 | pet | 4 | 420 | 0.00952 |
| is | 11 | 420 | 0.02619 | boxing | 4 | 420 | 0.00952 |
| and | 10 | 420 | 0.02381 | up | 4 | 420 | 0.00952 |
| in | 10 | 420 | 0.02381 | an | 4 | 420 | 0.00952 |
| for | 7 | 420 | 0.01667 | boxer | 4 | 420 | 0.00952 |
| his | 7 | 420 | 0.01667 | s | 3 | 420 | 0.00714 |
| he | 6 | 420 | 0.01429 | balboa | 3 | 420 | 0.00714 |

# Document Language Models

- Suppose we have a document *D*, with language model $\theta_D$
- We can use this language model to determine the probability of a particular sequence of text

- How? We multiple the probability associated with each term in the sequence!

# Document Language Models

## language model estimation (top 20 terms)

| *term* | $tf_{t,D}$ | $N_D$ | *P(term\|D)* | *term* | $tf_{t,D}$ | $N_D$ | *P(term\|D)* |
|---|---|---|---|---|---|---|---|
| a | 22 | 420 | 0.05238 | creed | 5 | 420 | 0.01190 |
| rocky | 19 | 420 | 0.04524 | philadelphia | 5 | 420 | 0.01190 |
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| and | 10 | 420 | 0.02381 | up | 4 | 420 | 0.00952 |
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- What is the probability given by this language model to the sequence of text “rocky is a boxer”?

# Document Language Models

## language model estimation (top 20 terms)

| *term* | $tf_{t,D}$ | $N_D$ | *P(term\|D)* | *term* | $tf_{t,D}$ | $N_D$ | *P(term\|D)* |
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- What is the probability given by this language model to the sequence of text “a boxer is a pet”?

# Document Language Models

## language model estimation (top 20 terms)

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| his | 7 | 420 | 0.01667 | s | 3 | 420 | 0.00714 |
| he | 6 | 420 | 0.01429 | balboa | 3 | 420 | 0.00714 |

- What is the probability given by this language model to the sequence of text “a boxer is a dog”?

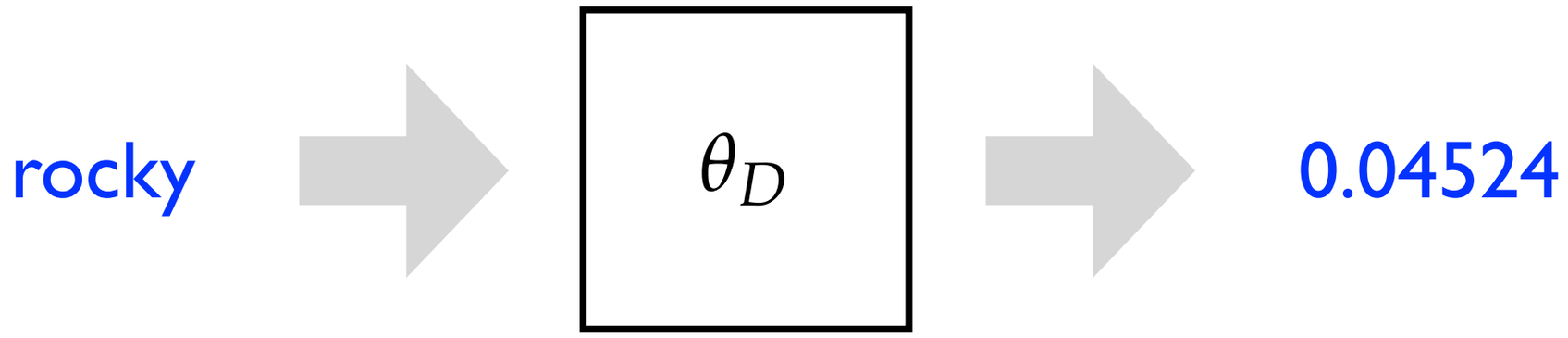
# Query-Likelihood Retrieval Model

- Objective: rank documents based on the probability that they are on the same topic as the query
- Solution:
  - Score each document (denoted by *D*) according to the probability given by its language model to the query (denoted by *Q*)
  - Rank documents in descending order of score

$$score(Q, D) = P(Q|\theta_D) = \prod_{i=1}^{n} P(q_i|\theta_D)$$

# Query-Likelihood Retrieval Model

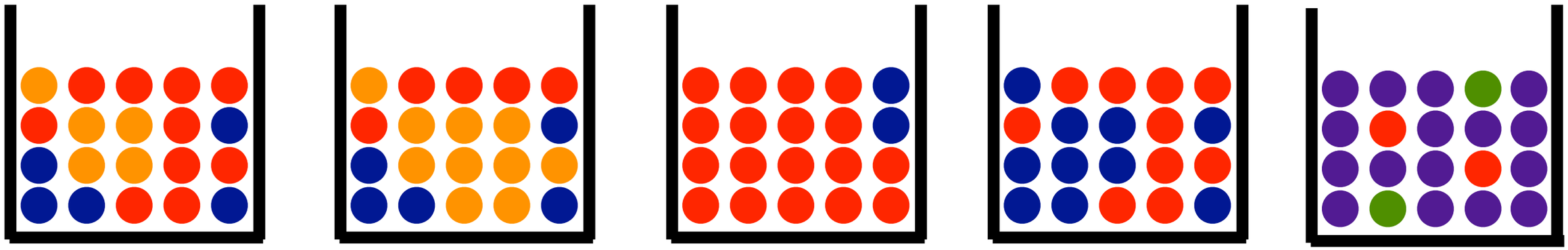
- Every document in the collection is associated with a language model
- Let $\theta_D$ denote the language model associated with document $D$
- You can think of $\theta_D$ as a "black-box": given a word, it outputs a probability




- Let $P(t|\theta_D)$ denote the probability given by $\theta_D$ to term ***t***

# Query-Likelihood Model

## back to our analogy

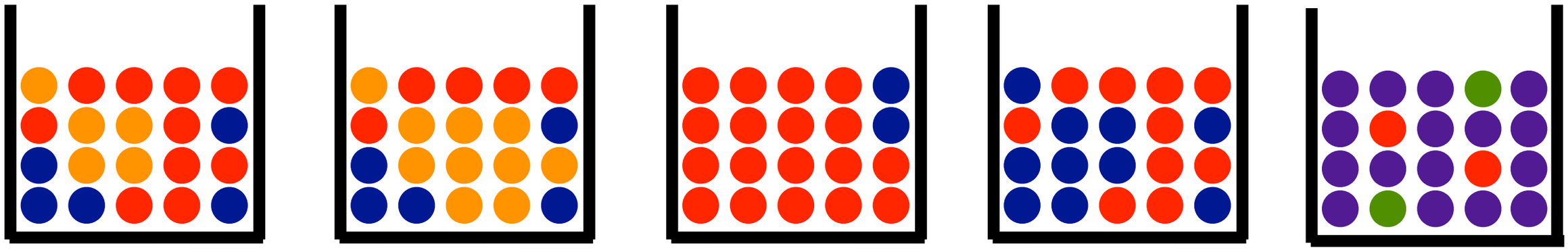


| $D_1$ | $D_2$ | $D_3$ | $D_5$ | $D_6$ |
|---|---|---|---|---|
| P(RED) = 0.50 | P(RED) = 0.25 | P(RED) = 0.90 | P(RED) = 0.50 | P(RED) = 0.10 |
| P(BLUE) = 0.25 | P(BLUE) = 0.25 | P(BLUE) = 0.10 | P(BLUE) = 0.50 | P(BLUE) = 0.80 |
| P(ORANGE) = 0.25 | P(ORANGE) = 0.50 | P(ORANGE) = 0.00 | P(ORANGE) = 0.00 | P(ORANGE) = 0.10 |

- Each document is scored according the probability that it "generated" the query
- What does it mean for a document to "generate" the query?

# Query-Likelihood Model

## back to our analogy



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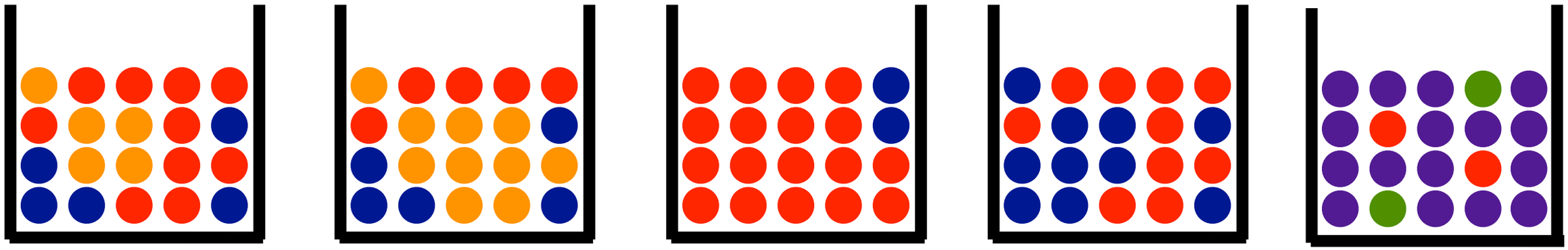
- Query = ●●●
- Which would be the top-ranked document and what would be its score?

# Query-Likelihood Model

## back to our analogy



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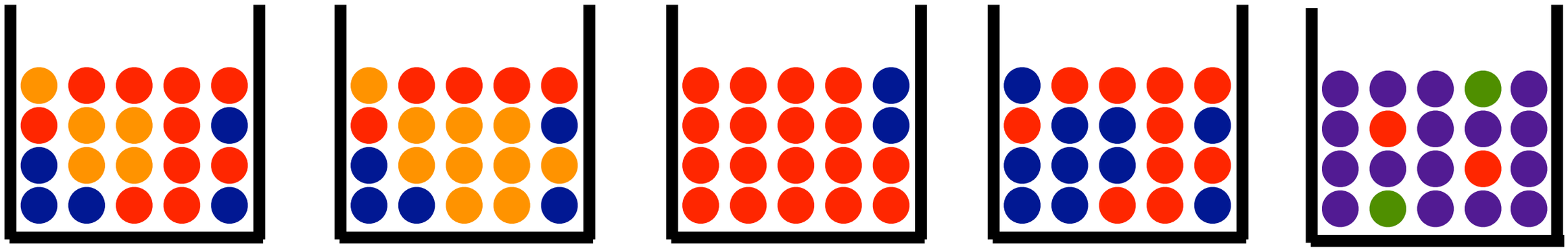
- Query = ●●
- Which would be the top-ranked document and what would be its score?

# Query-Likelihood Model

## back to our analogy



$D_1$

P(RED) = 0.50
P(BLUE) = 0.25
P(ORANGE) = 0.25

$D_2$

P(RED) = 0.25
P(BLUE) = 0.25
P(ORANGE) = 0.50

$D_3$

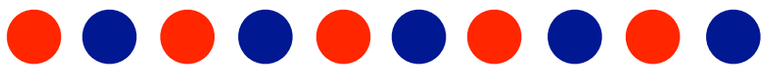
P(RED) = 0.90
P(BLUE) = 0.10
P(ORANGE) = 0.00

$D_5$

P(RED) = 0.50
P(BLUE) = 0.50
P(ORANGE) = 0.00

$D_6$

P(RED) = 0.10
P(BLUE) = 0.80
P(ORANGE) = 0.10

- Query = 
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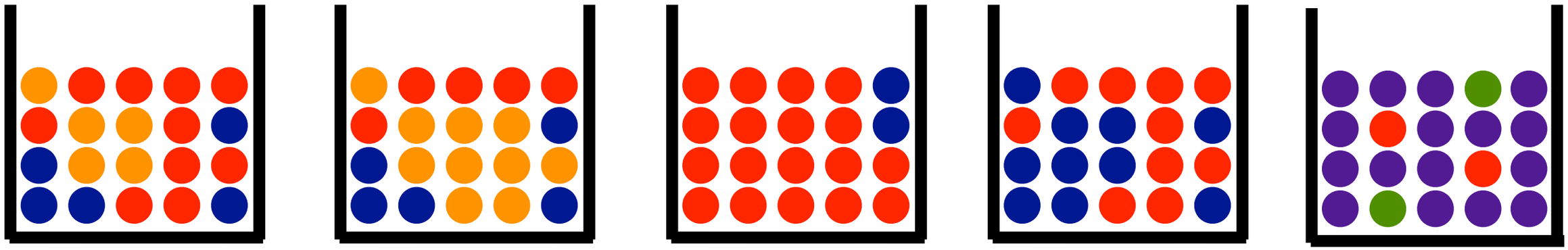
# Query-Likelihood Model

## back to our analogy



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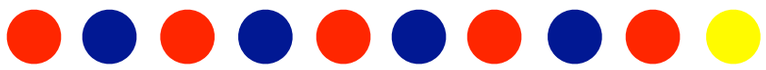
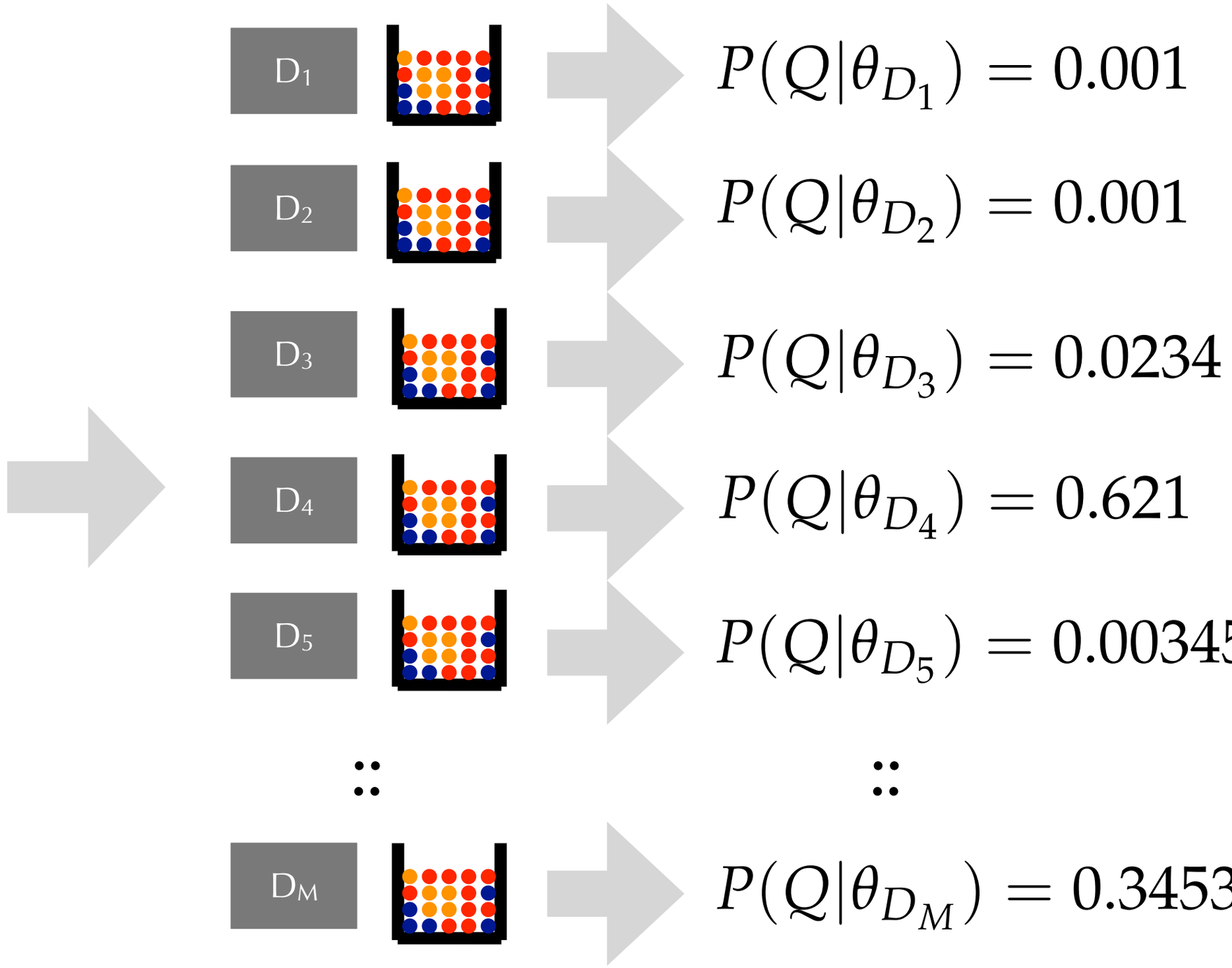
- Query = 
- Which would be the top-ranked document and what would be its score?

# Query-Likelihood Retrieval Model

$Q$

"rocky vs. apollo creed"



# Query-Likelihood Retrieval Model

$$score(Q, D) = P(Q|\theta_D) = \prod_{i=1}^{n} P(q_i|\theta_D)$$

$$score(\text{rocky vs apollo creed}, D_5) =$$

$$P(rocky|\theta_{D_5}) \times P(vs|\theta_{D_5}) \times P(apollo|\theta_{D_5}) \times P(creed|\theta_{D_5})$$

# Query-Likelihood Retrieval Model

- Because we are multiplying query-term probabilities, the longer the query, the lower the document scores (from all documents)

- Is this a problem?

# Query-Likelihood Retrieval Model

- Because we are multiplying query-term probabilities, the longer the query, the lower the document scores (from all documents)

- Is this a problem?
- No, because we're scoring documents for the same query

# Query-Likelihood Retrieval Model

$$score(Q, D) = P(Q|\theta_D) = \prod_{i=1}^{n} P(q_i|\theta_D)$$

- There are (at least) two issues with this scoring function
- What are they?

# Query-Likelihood Retrieval Model

- A document with a single missing query-term will receive a score of zero (similar to boolean AND)
- Where is IDF?

- Don’t we want to suppress the contribution of terms that are frequent in the document, but frequent in general (appear in many documents)?

# Outline

Introduction to language modeling

Language modeling for information retrieval

Query-likelihood Retrieval Model

Smoothing

Pseudo-relevance feedback and priors

# Smoothing Probability Estimates

- When estimating probabilities, we tend to ...
  - Over-estimate the probability of observed outcomes
  - Under-estimate the probability of unobserved outcomes
- The goal of smoothing is to ...
  - Decrease the probability of observed outcomes
  - Increase the probability of unobserved outcomes
- It's usually a good idea
- You probably already know this concept!

# Smoothing Probability Estimates

- YOU: Are there mountain lions around here?

- YOUR FRIEND: Nope.
- YOU: How can you be so sure?
- YOUR FRIEND: Because I've been hiking here five times before and have never seen one.

- YOU: ????

# Smoothing Probability Estimates

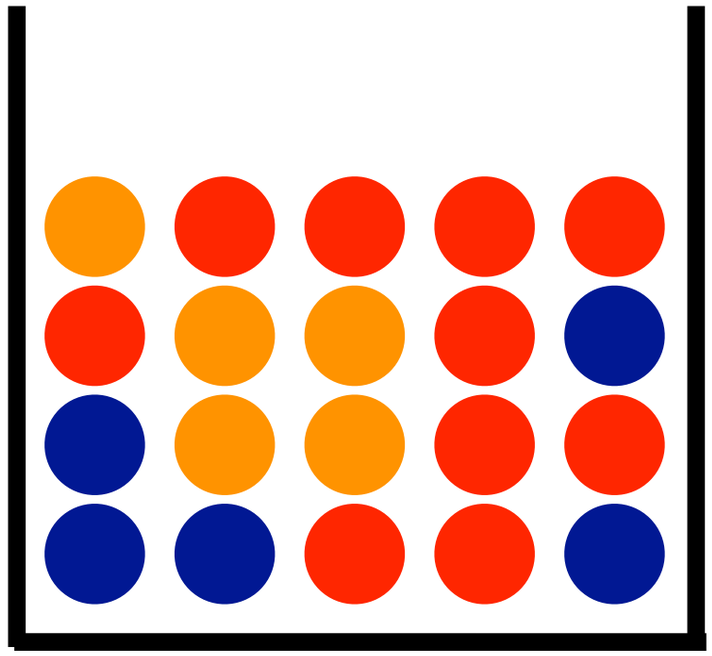
- YOU: Are there mountain lions around here?

- YOUR FRIEND: Nope.
- YOU: How can you be so sure?

- YOUR FRIEND: Because I've been hiking here five times before and have never seen one.
- MOUNTAIN LION: You should have learned about smoothing by taking INLS 509. Yum!

# Smoothing Probability Estimates

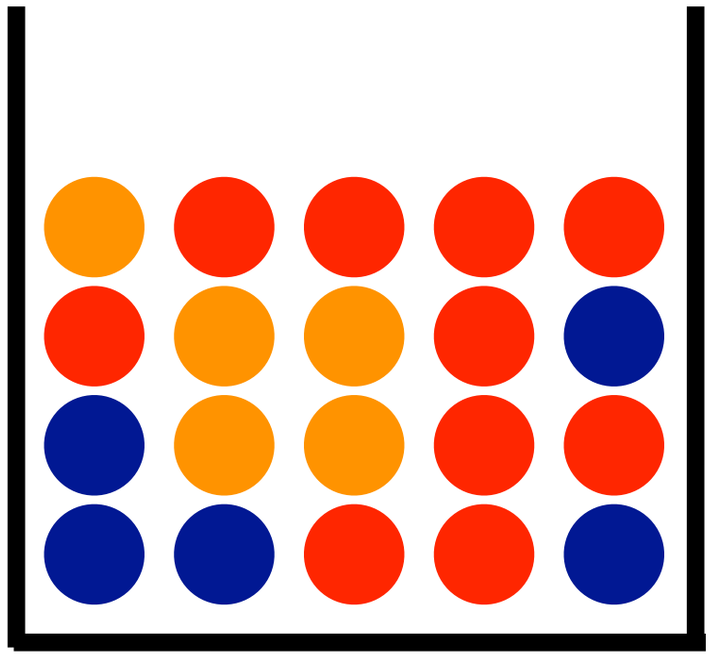
- Suppose that in reality this bag is a sample from a different, bigger bag ...
- And, our goal is to estimate the probabilities of that bigger bag ...
- And, we know that the bigger bag has red, blue, orange, yellow, and green balls.



P(RED) = 0.5
P(BLUE) = 0.25
P(ORANGE) = 0.25
P(YELLOW) = 0.00
P(GREEN) = 0.00

# Smoothing Probability Estimates

- Do we really want to assign YELLOW and GREEN balls a zero probability?
- What else can we do?

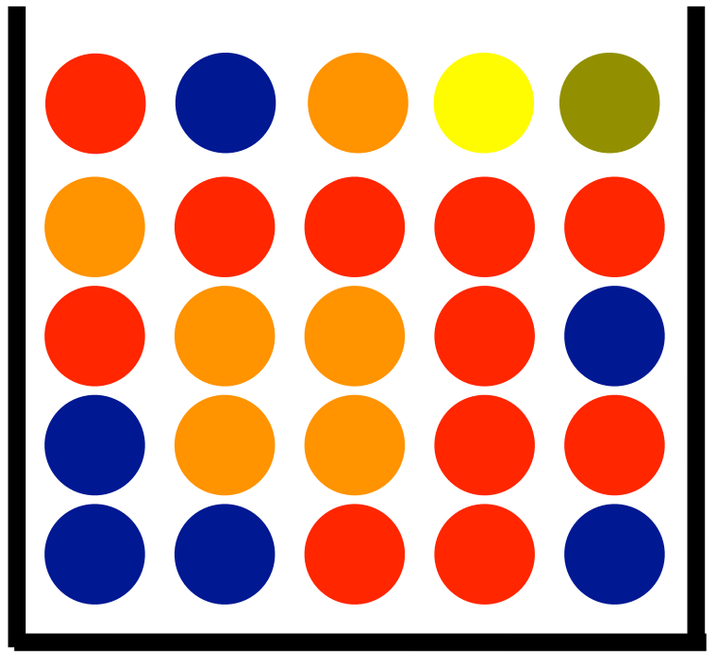


P(RED) = (10/20)
P(BLUE) = (5/20)
P(ORANGE) = (5/20)
P(YELLOW) = (0/20)
P(GREEN) = (0/20)

# Add-One Smoothing

- We could add one ball of each color to the bag
- This gives a small probability to unobserved outcomes (YELLOW and GREEN)
- As a result, it also reduces the probability of observed outcomes (RED, BLUE, ORANGE) by a small amount

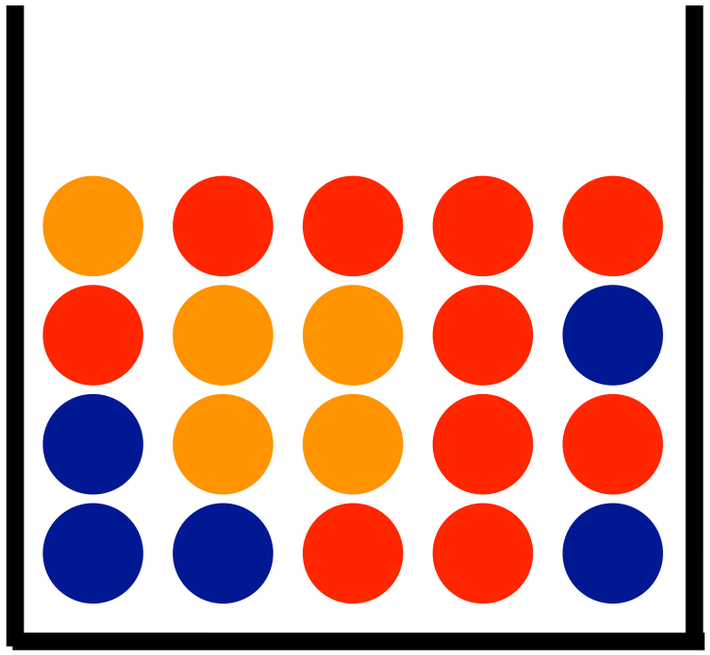
- Very common solution (also called ‘discounting’)



P(RED) = (11/25)
P(BLUE) = (6/25)
P(ORANGE) = (6/25)
P(YELLOW) = (1/25)
P(GREEN) = (1/25)

# Add-One Smoothing

- Gives a small probability to unobserved outcomes (YELLOW and GREEN) and reduces the probability of observed outcomes (RED, BLUE, ORANGE) by a small amount



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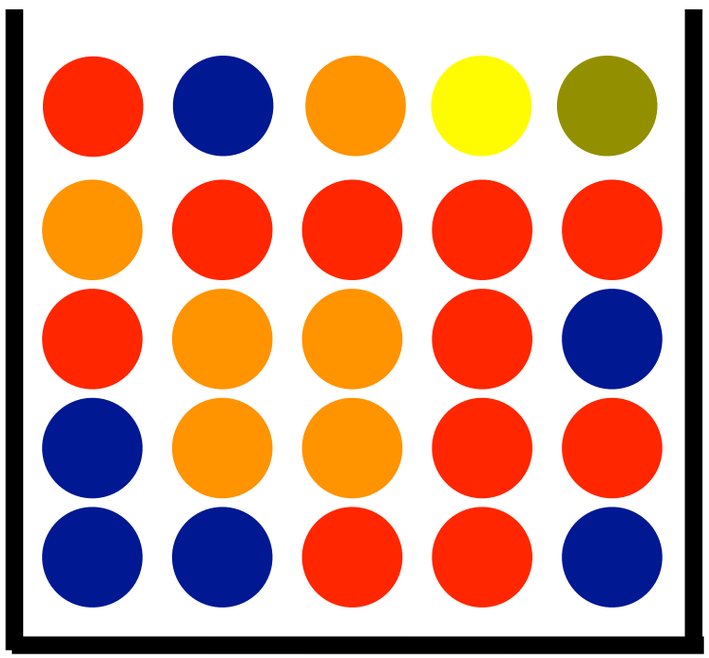

P(RED) = (11/25)
P(BLUE) = (6/25)
P(ORANGE) = (6/25)
P(YELLOW) = (1/25)
P(GREEN) = (1/25)

# Smoothing Probability Estimates

- Movie: Rocky (1976)
- Plot:

Rocky Balboa is a struggling boxer trying to make the big time. Working in a meat factory in Philadelphia for a pittance, he also earns extra cash as a debt collector. When heavyweight champion Apollo Creed visits Philadelphia, his managers want to set up an exhibition match between Creed and a struggling boxer, touting the fight as a chance for a "nobody" to become a "somebody". The match is supposed to be easily won by Creed, but someone forgot to tell Rocky, who sees this as his only shot at the big time. Rocky Balboa is a small-time boxer who lives in an apartment in Philadelphia, Pennsylvania, and his career has so far not gotten off the canvas. Rocky earns a living by collecting debts for a loan shark named Gazzo, but Gazzo doesn't think Rocky has the viciousness it takes to beat up deadbeats. Rocky still boxes every once in a while to keep his boxing skills sharp, and his ex-trainer, Mickey, believes he could've made it to the top if he was willing to work for it. Rocky, goes to a pet store that sells pet supplies, and this is where he meets a young woman named Adrian, who is extremely shy, with no ability to talk to men. Rocky befriends her. Adrain later surprised Rocky with a dog from the pet shop that Rocky had befriended. Adrian's brother Paulie, who works for a meat packing company, is thrilled that someone has become interested in Adrian, and Adrian spends Thanksgiving with Rocky. Later, they go to Rocky's apartment, where Adrian explains that she has never been in a man's apartment before. Rocky sets her mind at ease, and they become lovers. Current world heavyweight boxing champion Apollo Creed comes up with the idea of giving an unknown a shot at the title. Apollo checks out the Philadelphia boxing scene, and chooses Rocky. Fight promoter Jergens gets things in gear, and Rocky starts training with Mickey. After a lot of training, Rocky is ready for the match, and he wants to prove that he can go the distance with Apollo. The 'Italian Stallion', Rocky Balboa, is an aspiring boxer in downtown Philadelphia. His one chance to make a better life for himself is through his boxing and Adrian, a girl who works in the local pet store. Through a publicity stunt, Rocky is set up to fight Apollo Creed, the current heavyweight champion who is already set to win. But Rocky really needs to triumph, against all the odds...

# Smoothing Probability Estimates
## for document language models

- We can view a document as words sampled from the author's mind
- High-frequency words (e.g., rocky, apollo, boxing) are important
- Low-frequency words (e.g., shot, befriended, checks) are arbitrary
- The author chose these, but could have easily chosen others
- So, we want to allocate some probability to unobserved indexed-terms and discount some probability from those that appear in the document

# Smoothing Probability Estimates
## for document language models

- In theory, we could use add-one smoothing
- Do do this, we would add each indexed-term once into each document
  - Conceptually!
- Then, we would compute its language model probabilities
- In practice, a more effective approach to smoothing for information retrieval is called linear interpolation

# Linear Interpolation Smoothing

- Let $\theta_D$ denote the language model associated with document ***D***
- Let $\theta_C$ denote the language model associated with the entire collection
- Using linear interpolation, the probability given by the document language model to term ***t*** is:

$$P(t|D) = \alpha P(t|\theta_D) + (1 - \alpha)P(t|\theta_C)$$

# Linear Interpolation Smoothing

$$P(t|D) = \alpha P(t|\theta_D) + (1-\alpha) P(t|\theta_C)$$

the probability given to the term by the document language model

the probability given to the term by the collection language model

# Linear Interpolation Smoothing

$$P(t|D) = \alpha P(t|\theta_D) + (1-\alpha) P(t|\theta_C)$$

every one of these numbers
is between 0 and 1, so P(*t*|*D*)
is between 0 and 1

# Query Likelihood Retrieval Model
## with linear interpolation smoothing

- As before, a document's score is given by the probability that it "generated" the query
- As before, this is given by multiplying the individual query-term probabilities
- However, the probabilities are obtained using the linearly interpolated language model

$$score(Q, D) = \prod_{i=1}^{n} (\alpha P(q_i|\theta_D) + (1 - \alpha)P(q_i|\theta_C))$$

# Query Likelihood Retrieval Model
## with linear interpolation smoothing

- Linear interpolation helps us avoid zero-probabilities
- Remember, because we're multiplying probabilities, if a document is missing a single query-term it will be given a score of zero!
- Linear interpolation smoothing has another added benefit, though it's not obvious
- Let's start with an example

# Query Likelihood Retrieval Model

## no smoothing

- Query: apple ipad
- Two documents ($D_1$ and $D_2$), each with 50 term occurrences

| | $D_1$ $(N_{D1}=50)$ | $D_2$ $(N_{D2}=50)$ |
|---|---|---|
| apple | 2/50 = 0.04 | 3/50 = 0.06 |
| ipad | 3/50 = 0.06 | 2/50 = 0.04 |
| *score* | (0.04 x 0.06) = 0.0024 | (0.06 x 0.04) = 0.0024 |

# Query Likelihood Retrieval Model
## no smoothing

- Query: apple ipad
- Two documents ($D_1$ and $D_2$), each with 50 term occurrences

| | $D_1$ $(N_{D1}=50)$ | $D_2$ $(N_{D2}=50)$ |
|---|---|---|
| apple | 2/50 = 0.04 | 3/50 = 0.06 |
| ipad | 3/50 = 0.06 | 2/50 = 0.04 |
| *score* | (0.04 x 0.06) = 0.0024 | (0.06 x 0.04) = 0.0024 |

- Which query-term is more important: apple or ipad?

# Query Likelihood Retrieval Model
## no smoothing

- A term is descriptive of the document if it occurs many times in the document
- But, not if it occurs many times in the document and also occurs frequently in the collection

# Query Likelihood Retrieval Model

## no smoothing

- Query: apple ipad
- Two documents ($D_1$ and $D_2$), each with 50 term occurrences

| | $D_1$ $(N_{D1}=50)$ | $D_2$ $(N_{D2}=50)$ |
|---|---|---|
| apple | 2/50 = 0.04 | 3/50 = 0.06 |
| ipad | 3/50 = 0.06 | 2/50 = 0.04 |
| *score* | (0.04 x 0.06) = 0.0024 | (0.06 x 0.04) = 0.0024 |

- Without smoothing, the query-likelihood model ignores how frequently the term occurs in general!

# Query Likelihood Retrieval Model
## with linear interpolation smoothing

- Suppose the corpus has 1,000,000 term-occurrences
- apple occurs 200 / 1,000,000 times
- ipad occurs 100 / 1,000,000 times
- Therefore:

$$P(\text{apple}|\theta_C) = \frac{200}{1000000} = 0.0002$$

$$P(\text{ipad}|\theta_C) = \frac{100}{1000000} = 0.0001$$

# Query Likelihood Retrieval Model

## with linear interpolation smoothing

$$score(Q, D) = \prod_{i=1}^{n} (\alpha P(q_i|\theta_D) + (1 - \alpha) P(q_i|\theta_C))$$

| | $D_1$ ($N_{D1}$=50) | $D_2$ ($N_{D2}$=50) |
|---|---|---|
| *P(apple\|D)* | 0.04 | 0.06 |
| *P(apple\|C)* | 0.0002 | 0.0002 |
| *score(apple)* | 0.0201 | 0.0301 |
| *P(ipad\|D)* | 0.06 | 0.04 |
| *P(ipad\|C)* | 0.0001 | 0.0001 |
| *score(ipad)* | 0.03005 | 0.02005 |
| *total score* | 0.000604005 | 0.000603505 |

$$\alpha = 0.50$$

# Query Likelihood Retrieval Model
## with linear interpolation smoothing

- Linear interpolation smoothing does not only avoid zero probabilities ...
- It also introduces an IDF-like scoring of documents
  - terms are that are less frequent in the entire collection have a higher contribution to a document's score
- Yes, but we've only seen an example. Where is the mathematical proof!?

# Query Likelihood Retrieval Model

## with linear interpolation smoothing

$$
\begin{aligned}
p(q \mid d) &= \prod_{q_i \in q} p(q_i \mid d) \\
&= \prod_{q_i \in q} \left(\lambda p_{MLE}(q_i \mid d) + (1-\lambda) p_{MLE}(q_i \mid C)\right) && \textbf{Mixture model} \\
&= \prod_{q_i \in q} \left(\lambda p_{MLE}(q_i \mid d) + (1-\lambda) p_{MLE}(q_i \mid C)\right) \frac{(1-\lambda) p_{MLE}(q_i \mid C)}{(1-\lambda) p_{MLE}(q_i \mid C)} && \textbf{Multiply by 1} \\
&= \prod_{q_i \in q} \left( \left( \frac{\lambda p_{MLE}(q_i \mid d)}{(1-\lambda) p_{MLE}(q_i \mid C)} + 1 \right) (1-\lambda) p_{MLE}(q_i \mid C) \right) && \textbf{Recombine} \\
&= \prod_{q_i \in q} \left( \frac{\lambda p_{MLE}(q_i \mid d)}{(1-\lambda) p_{MLE}(q_i \mid C)} + 1 \right) \prod_{q_i \in q} (1-\lambda) p_{MLE}(q_i \mid C) && \textbf{Recombine} \\
&\propto \prod_{q_i \in q} \left( \frac{\lambda p_{MLE}(q_i \mid d)}{(1-\lambda) p_{MLE}(q_i \mid C)} + 1 \right) && \textbf{Drop constant}
\end{aligned}
$$

**"tf"**

**"idf"**

(slide courtesy of Jamie Callan)